Radio Polarimetry as a Probe of Interstellar Magnetism

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Chapter 1

Introduction

1.1 Magnetic Fields - A Historical Perspective

Magnetic fields are ubiquitous in the Universe. They have been detected in over 100 galaxies (Widrow, 2002), including in elliptical, spiral, barred and irregular galaxies and also throughout galaxy clusters such as the Coma supercluster complex (Kim et al., 1990). Not only have magnetic fields been measured on the largest extragalactic scales, but magnetism is also seen on small scales, in stars and planets.

While the origin of magnetism is still to be discovered and is one of the key science questions for the next generation of telescopes, humans have known and discussed the phenomenon for many centuries. The ancient Greek philosopher Thales (625 - c. 545 BCE) is generally accredited with being the first person to investigate magnetic fields. Although none of his works survived ancient times, Aristotle mentions him in his work “De Anima”: “And Thales, according to what is related of him, .... said that the lodestone has a soul because it moves iron”. However, it was the ancient Chinese people who first developed a use for magnetism by inventing the compass. The first record of a compass being used in China was in 1088CE by Shen Kuo.

The study of modern magnetism began in 1600CE but it wasn’t until 1908 that the first magnetic fields beyond Earth were detected. Hale (1908) used an optical telescope and the Zeeman effect to measure magnetic fields in sunspots. Since then, our understanding of celestial magnetic fields has increased and we now know that magnetic fields play a role in determining the evolution of galaxies and galaxy clusters; are responsible for exerting pressure and influencing the behaviour of interstellar gas and cosmic rays in the interstellar medium (ISM); and have a significant role in star formation. However, many questions remain, including: what is the strength and structure of magnetic fields in the intergalactic medium (IGM), does the structure of the early Universe map to the magnetic field formation in the same period and what is the structure and orientation of the magnetic field in our own Galaxy?
1.2 Measurement of Magnetic Fields

Interstellar magnetic fields can be observed indirectly at ultra-violet (UV), optical, infrared (IR), microwave and radio wavelengths.

Following the first detection of a celestial magnetic field a century ago, there were no advances for 40 years until the detection of magnetic fields in Ap stars by Babcock (1947). Since that date 62 years ago, there has been good progress in detecting and understanding cosmic magnetic fields. Historically, optical and radio astronomy were used to study magnetism. However, with the increased technical capabilities of many telescopes, sub-mm and IR polarimetry has become another valuable option. In this section we shall briefly describe the optical, infrared, sub-mm and mm milestones for studying magnetic fields before considering in depth the achievements of radio astronomy.

1.2.1 Optical Polarisation of Starlight

Optical polarisation of starlight was first imaged by Hall and Mikesell (1949) and by Hiltner (1949). Both authors found that the polarised light from nearby stars had similar orientation, indicating that the polarizing mechanism was a source other than the individual stars. Shortly after these papers were published, Davis and Greenstein (1951) published a theory on the origin of polarised starlight. They proposed that an elongated dust grain, spinning about its shortest axis would have a preferred orientation when located in a magnetic field. Material that has unpaired electrons is paramagnetic and in the presence of an external magnetic field, such as the interstellar magnetic field, the unpaired electrons are all oriented in the same direction, causing grain magnetization. In the case of an elongated grain, if the shortest axis is aligned with the direction of the magnetic field, then the grains would absorb light polarised along the long axis of the grain which is perpendicular to the field. This results in the transmitted radiation having a polarisation direction parallel to the magnetic field.

The above mechanism has been modified over the years (Jones and Spitzer, 1967; Purcell, 1979) to account for observations but it is still believed to be responsible for most of the grain alignment (Goodman, 1996). Polarisation of starlight is most useful for detecting magnetic fields in the Solar neighbourhood out to about 1 – 3 kpc (Han, 2008). Early observers catalogued the polarisation of stars in the Northern hemisphere, where the work was compiled by Behr (1959). A comparison catalogue for the Southern hemisphere was completed by Mathewson and Ford (1970) who combined their results with surveys in the Northern Hemisphere to produce the first all-sky map of galactic polarisation, with 1800 stars. Their general conclusion was that the magnetic field is aligned with the Galactic plane. Since this first catalogue, many others have been compiled using successively more stars. The most recent catalogues (Heiles, 2000; Berdyugin et al., 2001) confirm the hypothesis of Mathewson and Ford (1970). Recent analyses (Heiles, 1996b; Fosalba et al., 2002) show that the local magnetic field of our Galaxy is directed towards Galactic longitude $l \sim 82^\circ$ and appears to follow the local spiral arm. Fig. 1.1 shows the polarised starlight of nearby stars (top panel) and more distant stars (bottom panel). The structure in the bottom panel is much smoother than in the top panel because of
1.2. Measurement of Magnetic Fields

Figure 1.1 These two plots show different distributions of optically polarised starlight. The top plot shows the magnitude and orientation of the polarisation of nearby stars while the bottom plot shows the polarisation of stars at distances > 1kpc. The stars further away have a magnetic field parallel to the Galactic plane. The polarisation from the nearby stars appears more turbulent and chaotic but this may be solely due to due to the lack of averaging that occurs in the polarisation from stars at small distances. This figure is reproduced from Fosalba et al. (2002)

averaging effects (Goodman, 1996).

1.2.2 Polarisation Observations at other Wavelengths (IR, mm)

Polarisation studies in the IR waveband of the electromagnetic spectrum were initiated to investigate the magnetic fields within the densest regions of dark clouds (Vrba et al., 1976). It was thought that near-IR observations would produce results from regions deep inside the optically opaque dark clouds (Goodman, 1996). However, Goodman et al. (1992) discovered that cold dark clouds were inefficient polarizers of background starlight since the polarisation angle maps had identical mean and dispersion distributions in both the optical and near-IR, implying that a similar polarizing mechanism occurs in both wavebands.

At longer wavelengths, from mid-IR to mm, scattering effects are negligible and the thermal emission from dust should be partially linearly polarised (Stein, 1966). A particular characteristic of the mid-IR is that the linear polarisation due to the aligned grains
can be caused by either emission and/or absorption and these processes separately yield a polarisation angle that differs by 90° for a particular grain alignment (Aitken et al., 2004). For example, the absorptive polarisation near 10 µm probes the line-of-sight averaged magnetic field for embedded luminous sources, while the emissive component yields the direction of the field local to the source. In order to separate emissive and absorptive polarisation effects, spectropolarimetric observations are required (Aitken, 1996) and the wavelength-dependent polarisation angle information is used.

The earliest positive detection of far-IR (FIR) polarisation in the Galaxy was by Cudlip et al. (1982) who used a balloon-borne telescope to make measurements of the HII region M42. Other early work (Hildebrand et al., 1984; Dragovan, 1986) focused on molecular clouds. One problem with studying FIR polarisation from ground-based telescopes is the limited windows where the Earth’s atmosphere is transparent to radiation in these frequency ranges. The windows fall at 350 µm, 450 µm, 750 µm, 850 µm and 1.3 mm. Recent improvements in instrumentation and space-based telescopes such as SOFIA (Stratospheric Observatory for Infrared Astronomy) (Becklin, 1997), have opened the field of FIR polarimetry (Dotson et al., 2000). Emission at longer wavelengths (50 – 30 µm) comes from colder dust (10 – 50 K), while the warmer dust (100 – 500 K) (Whittet, 2003) emits at shorter wavelengths. Hence it is possible to observe polarisation signals coming from different regions in the same cloud complex (Schleuning et al., 2000). In most cases, polarisation maps at different wavelengths in the IR spectrum indicate a common magnetic field orients the dust grains. Polarimetry can be used to better understand grain properties and their role in cloud complexes (Lazarian et al., 1997; Whittet, 2003).

Dust may be a major contributor to the observed Galactic emission at microwave frequencies (λ ∼ 3 – 300 mm). At these frequencies the thermal radiation from dust is weak and the dust emission comes principally from electric dipoles (Draine and Lazarian, 1998) or magnetic dipoles (Draine and Lazarian, 1999). Understanding Galactic dust emission may be significant for Cosmic Microwave Background (CMB) studies where multiwavelength polarimetry from both absorption and emission mechanisms is used to calculate magnetic fields and dust grain properties. Understanding the Galactic foreground contribution is critical to unraveling the structure of the polarised CMB.

### 1.2.3 Zeeman Effect

The Zeeman effect is named for Dutch physicist Pieter Zeeman who noticed that the yellow sodium doublet lines broadened in an applied magnetic field (Zeeman, 1897). The effect was explained classically by Lorentz (1899) but a more modern description comes from quantum mechanics. There exists degeneracies between the various electron energy levels that are broken when a magnetic field is applied, producing several closely spaced spectral lines.

In an astrophysical context, a magnetic field produces small frequency shifts in the left and right circularly polarised components of a given spectral line with respect to the intrinsic central frequency of the atom or molecule. As mentioned in Section 1.1 the first measurement of a celestial magnetic field was undertaken using Zeeman splitting.

Zeeman splitting is a powerful tool as the magnetic field can be directly determined
from the energy difference between the electron levels. The separation in energy of the magnetic hyperfine levels from the unsplit level of the zero magnetic field case is given by

\[ \Delta E = -\mu_B m_F g B, \]

where \( \Delta E \) is the difference in energy levels, \( \mu_B \) is the Bohr magneton, \( m_F \) is the quantum number of the splitting, \( B \) is the magnetic field strength, and \( g \) is the Landé \( g \)-factor, which is more fully explained in other documents, such as Rybicki and Lightman (1986). However, the effect is difficult to observe as the shift (\( \Delta \nu \)) in the spectral lines is small:

\[ \Delta \nu = \frac{\mu_B m_F g B}{\hbar}, \]

where \( \hbar \) is Planck’s constant. The frequency shift \( \Delta \nu \) is usually much smaller than the observed line width, making Zeeman splitting observations resolution limited (Heiles and Crutcher, 2005). In radio astronomy, the Zeeman effect can sometimes be detected by looking at the circular polarization, represented by the Stokes \( V \) parameter. Stokes \( V \) is related to the line-of-sight magnetic field in the following manner:

\[ V(\nu) \propto B_\parallel \times \frac{dI}{d\nu}. \]

The splitting of components is very small for diffuse gas. In denser regions such as molecular clouds the search was successful. Although Crutcher and Kazes (1983) took 13 years to confirm the first OH Zeeman detection of Turner and Verschuur (1970). More recent detections are summarised in Crutcher (1999) and Bourke et al. (2001) and the Zeeman effect in OH has now become the primary method for measuring magnetic field strengths in molecular clouds. While studies showed structure in the magnetic field, the small number of sources made it impossible to determine the overall coherent structure (Roberts et al., 1995).

Lower frequencies produce better Zeeman splitting results because, although the splitting is independent of the line frequency itself, the higher the line frequency, the broader the line width. There are exceptions and CN at 114 GHz (Crutcher, 1999) and \( \text{C}_2\text{H} \) at 87.4 GHz (Bel and Leroy, 1998) are examples of molecules which show significant splitting. The molecular lines which split at higher frequencies are valuable as they trace denser environments, often obscured at other wavebands. Although Zeeman detection experiments are very time intensive, the advent of telescopes such as ALMA (Crutcher and Troland, 2008) will enable magnetic field determinations in areas not previously scrutinised.

### 1.3 Polarisation at Radio Wavelengths

#### 1.3.1 Emission Mechanism

Synchrotron radiation from the Galaxy was the first emission detected in radio astronomy (Jansky, 1933), although it was not until the 1950s when detailed radio maps of the Galaxy were produced, that the connection was made between radio emission and the synchrotron
mechanism. Le Roux (1961) gives a full derivation of synchrotron radiation and Ginzburg and Syrovatskii (1965) discuss earlier work involving synchrotron radiation.

Cosmic rays (high energy nuclei and electrons) are an important component of the ISM along with magnetic fields and interstellar gas. While the origin of cosmic rays is still not comprehensively understood, it is believed (Osterbrock and Ferland, 2006; Yoshida, 2008) that the electrons with an energy approaching 100 - 1000 TeV have been accelerated in supernova remnants (SNRs) (Koyama et al., 1995), suggesting this is one mechanism for the primary formation of cosmic rays.

When cosmic ray electrons, traveling at relativistic speeds, are deflected in a magnetic field, synchrotron radiation is emitted. For a single electron, this can be expressed (Rybicki and Lightman, 1986) as:

$$\nu \sim \Gamma^2 \frac{eB}{2\pi m_0},$$

(1.4)

where \(m_0\) is the rest mass of a relativistic particle in a magnetic field \(B\) with a Lorentz factor \(\Gamma\); \(\nu\) is the critical frequency for the synchrotron radiation. The graphical representation of synchrotron radiation, depicted in Fig. 1.2, shows that the velocity vector of the particle generates the surface of a cone, with the synchrotron emission beamed within the surface of the cone, at an angle of:

$$\theta = \pm \frac{m_0 c^2}{E}.$$  

(1.5)

On the surface of the cone the radiation is 100% linearly polarised with its electric field in the direction \(-v \times B\), where \(v\) is the direction of propagation of the electron.

For an ensemble of electrons with a distribution of energies and a uniform distribution of pitch angle, relative to the magnetic field direction, the distribution may be expressed as a power law (Rybicki and Lightman, 1986):

$$n(E) dE = n_0 E^{-\gamma} dE,$$

(1.6)
where \( n(E) \) is the number of electrons with energies between \( E \) and \( E + dE \), \( n_0 \) is a normalising electron number density and \( \gamma \) is a constant power law index. Assuming a uniform magnetic field, the total emission intensity \( I(\nu) \) is given by:

\[
I(\nu) \propto n_{\text{CR}}B_{\perp}^{(\gamma+1)/2}\nu^{-\frac{(\gamma+1)}{2}} \propto (B_0\sin \theta)^{(\gamma+1)/2} \nu^{-\frac{(\gamma+1)}{2}},
\]

(1.7)

where \( n_{\text{CR}} \) is the number density of electrons in the emitting volume, \( B_{\perp} \) is the magnetic field component perpendicular to the line of sight and \( \gamma \) is directly related to the observed spectral index \( \alpha = -\frac{\gamma + 1}{2} \), which describes the dependence of the measured intensity on frequency\(^1\).

The degree of linear polarisation, \( p \), is independent of frequency and depends on the spectral index of the cosmic ray electrons:

\[
p(\alpha) = \frac{3 - 3\alpha}{5 - 3\alpha},
\]

(1.8)

and for a spectral index of \( \alpha = -0.7 \), \( p \approx 72\% \). This high level of polarisation is usually not observed because of depolarisation effects as described in Chap. 3.

With the construction of large single dish radio telescopes such as Parkes and Effelsberg the global distribution of radio emission was observed. The all-sky map by Haslam et al. (1981, 1982) (see Fig. 1.3) shows the radio sky at 408 MHz.

Beuermann et al. (1985) used this map to model the diffuse synchrotron background of the Galaxy. The Beuermann model is a thick synchrotron disk with a thinner disk embedded, both disks exhibiting spiral structure. The diffuse Galactic emission is concentrated close to the Galactic Plane but also extends out to high latitudes. Discrete synchrotron sources, such as supernova remnants (SNRs) have a narrow distribution about the Galactic Plane and strongly influence the ISM.

### 1.3.2 The Magnetised ISM

Supernova explosions, stellar winds and photons all contribute to power a complex and non-linear system of gas, dust, magnetic fields and cosmic rays that make up the ISM. Magnetic fields, cosmic rays and baryonic matter exert approximately equal pressure and are bound together by electromagnetic forces. Interstellar matter accounts for \( \sim 10 - 15\% \) of the total mass of the Galactic disk. The ISM is denser near the Galactic Plane and along the spiral arms, while being very inhomogeneously distributed at small scales. The composition of interstellar matter is roughly similar to what is found in the Sun (90.8% H, 9.1% He and 0.12% heavier elements). The material exists in a number of forms: molecular, cold atomic, warm atomic, warm ionised and hot ionised gas. This thesis is mainly concerned with ionised gas and the magneto-ionic medium, with little discussion of neutral or cold matter. Apart from the densest parts of molecular clouds, whose degree of ionisation is exceedingly low, virtually all interstellar regions are sufficiently ionised for their neutral component to remain tightly coupled to the charged component and hence to the local magnetic field.

\(^1I(\nu) \propto \nu^\alpha\)
As expected the emission is concentrated along the Galactic Plane. However, the feature known as Loop I, is clearly arching up from $l = 55^\circ$ towards the North Galactic Pole. This figure is adapted from Haslam et al. (1981).

The Warm Ionised Medium (WIM)

The UV radiation from the hottest, most massive stars in the Galaxy (OB-stars) is responsible for ionising most of the hydrogen around such stars. The ionised clouds are known as HII regions and their boundaries are determined by volume in which the rate of UV photoionisation equals the rate of recombination of electrons. When free electrons within an HII region pass near a positive ion ($\text{H}^+$, $\text{He}^+$) they are accelerated by the Coulomb field and emit radiation known as “free-free” or bremsstrahlung emission. The average electron density of an HII region is $\sim 10^3\text{cm}^{-3}$.

As well as in HII regions, gas in the WIM, at $T \sim 8000\text{K}$, has been found to be more widely distributed. The average electron density of the gas in the WIM is $0.2 – 0.5 \text{ cm}^{-3}$ (Ferrière, 2001). Struve and Elvey (1938) were the first to detect the diffuse gas. The ionised gas has mostly been mapped using the Hα line, and panoramic studies include the Sivan (1974) Hα photographic survey, the Wisconsin Hα mapper (WHAM) (Reynolds et al., 1999; Haffner et al., 2003) and the Virginia Tech Spectral line Survey (VTSS) (Dennison et al., 1999) in the north and the Southern H-Alpha Sky Survey Atlas (SHASSA) (Gaustad et al., 2001) and the SuperCOSMOS survey done with the Anglo-Australian Observatory (Parker et al., 2005) in the south. In 2009, a southern version of
1.3. Polarisation at Radio Wavelengths

WHAM will begin (Madsen, private communication).

One limitation of the Hα line as a probe comes from the obscuration caused by dust, which restricts visibility to within 2 – 3 kpc of the Sun. Nevertheless, studies of HI regions and diffuse gas show that most of the ionised gas is distributed within 1 kpc of the Galactic Plane, with some higher latitude extended structures (Reich, 2006). Pulsar dispersion measures (obtained by measuring the pulse arrival time as a function of wavelength), can also be used to probe the ionised ISM. Pulsar distances, dispersion measures and scattering effects were used by Taylor and Cordes (1993) and Cordes and Lazio (2002) to model the thermal electron distribution for the Galaxy. A recent study (Gaensler et al., 2008) of the WIM showed that the scale height of free electrons in the diffuse WIM is \(1830^{+120}_{-250}\) pc, double the previously accepted value.

**The Hot Ionised Medium (HIM)**

Spitzer (1956) was the first to suggest that there might be hot ionised gas at \(T \sim 10^5 – 10^6\) K in the Galaxy. This hot interstellar gas is believed to have been generated mainly by supernovae (SNe) and stellar winds from massive stars (McCray and Snow, 1979; Spitzer, 1990; Joung and Mac Low, 2006). The hot gas forms when the shock wave from a SN sweeps through the ISM. Cox and Smith (1974) showed that a SN rate of 1 per 50 years would cause these bubbles of hot gas to overlap and form a network of interconnecting tunnels. McKee and Ostriker (1977) used this information to form a self-consistent model of the ISM - the three-phase model. However, this model fails to predict the volume filling factors of the diffuse ionised gas and the HIM, as was shown in McCray and Snow (1979) and Heiles (1987), partly because supernovae tend to be clustered in regions to create superbubbles.

The Sun is believed to be located inside one of these SN bubbles (Cox and Reynolds, 1987), discovered as an under-dense region in starlight reddening maps (Fitzgerald, 1968). The gas inside the bubble has a temperature of \(\sim 10^6\) K and a density of \(0.001 \text{ cm}^{-3}\). The origin of our local bubble (LB) was postulated to have been at least one SN (Snowden et al., 1990), or perhaps multiple events (Smith and Cox, 2001). Maíz-Apellániz (2001) claims that our LB can be traced to an OB association, supporting the Smith and Cox theory of a multiple SN origin. Within the LB there are a number of smaller cloudlets and the Sun is found within one of these, the Local Interstellar Cloud (LIC) (Lallement and Bertin, 1992). As the density inside the LB is low, it is very difficult to directly measure the local magnetic field. Pulsar dispersion measures indicate that the uniform component of the magnetic field near the Sun is \(B \sim 1.4 \mu\text{G}\) with correlation lengths \(\sim 100\) pc (Rand and Kulkarni, 1989).

Very weak interstellar polarisation caused by magnetically aligned dust grains has been observed towards stars within \(\sim 35\) pc of the Sun (Tinbergen, 1982). Frisch (2005) found that when pre-aligned grains approach the heliosphere, the gas densities are too low to disrupt the alignment so the polarisation detected nearby should indicate the direction of the magnetic field at the heliosphere. The local maxima of polarisation are located at ecliptic longitudes that are offset by \(\sim 35^\circ\) from the alignment direction of largest dust grains flowing into the heliosphere and from the gas upwind direction (Frisch
et al., 1999), which suggests that the local ISM magnetic fields are wrapped around the heliosphere (Frisch, 2007).

### 1.3.3 Polarisation Studies of the Magnetised ISM

#### The Milky Way

The most detailed studies of the magnetised ISM are made in our own Galaxy, the Milky Way. We can determine the properties of magnetic fields on many length scales, from sub-pc to many kpcs. Of course, because of our location in the Milky Way there are frequently problems with geometric distortions, line-of-sight juxtapositions, and difficulties in obtaining an overview of the large scale magnetic field. However, it is hoped that the next generation radio telescopes, such as the LOw Frequency ARray (LOFAR), Murchison Wide-field Array (MWA), the Australian SKA Pathfinder (ASKAP) and the Square Kilometre Array (SKA), will image other galaxies in the same detail as we have with the Milky Way, and we can use those models from external galaxies to explain the structure of our Galaxy.

The total magnetic field strength in the local Milky Way is believed to be about $6\mu$G (Strong et al., 2000). The strength of the field can be determined from the total intensity of synchrotron emission, assuming equipartition between the energy densities of the magnetic field and the whole population of cosmic rays, with a ratio $K$ between the numbers of cosmic ray protons and electrons in the relevant energy range (usually $K \approx 100$). Strong et al. (2007) independently confirmed the $6\mu$G result by direct measurements of the cosmic ray density near the Sun, with their radial distribution being inferred from gamma-ray data.

The total magnetic field strength is believed to be as high as $10\mu$G in the inner part of the Galaxy (Beck, 2001). In non-thermal filaments near the Galactic centre it is claimed that the field strength may be as high as several hundred $\mu$G (Yusef-Zadeh et al., 1996), but the large-scale diffuse field in the Galactic Centre region is much weaker (Novak, 2005).

Synchrotron polarisation observations in the solar neighbourhood indicate that the ratio of the strength of the large-scale and total magnetic fields $\langle B_{\text{reg}}/B_{\text{tot}} \rangle$ is approximately 0.6 (Berkhuijsen, 1971; Brouw and Spoelstra, 1976; Heiles, 1996a). This is similar to the value estimated from the total synchrotron emission observed along the local spiral arm (Phillipps et al., 1981). However, this value for the ratio would make the local regular magnetic field strength about $4\mu$G, which is more than double the value ($1.4\mu$G) measured using pulsar rotation measures (RMs) and dispersion measures (Rand and Lyne, 1994; Han and Qiao, 1994). In the inner Norma arm, the average strength of the coherent magnetic field is $4.4\pm0.9\mu$G (Han, 2002). Han et al. (2006) claim that the ordered field is stronger in the Galactic arms than in the interarm regions. However, this result is contrary to findings by Brown et al. (2007) with their studies of the magnetic fields in the 4th quadrant of the Galaxy.

Depending on the telescope, radio polarisation observations can observe structures on all scales in the Galaxy. On smaller scales, the Galactic magnetic field has a significant
turbulent component with an average strength of $\sim 5\mu G$ (Rand & Kulkarni, 1989). The turbulent component has the same order of magnitude as that found in the galaxy NGC 6946 (Beck et al., 1999). Other structures in polarised emission carry valuable information about the turbulence in the Galaxy, which are described in later chapters of this thesis.

**External Galaxies**

Magnetic fields have been mapped in many galaxies. Radio polarisation observations at the shorter wavelengths ($\lambda \leq 10\text{cm}$) are only marginally affected by Faraday rotation, so that polarisation angle maps show the inherent magnetic field structure. These maps have shown large scale spiral fields in spiral galaxies, as expected, but also in barred galaxies, flocculent galaxies (Soida et al., 2002) and even some irregular galaxies (Beck, 2005). In large spiral galaxies polarisation observations have shown that the spiral magnetic field structure generally does not align with the optical spiral arms. In some regions the polarised emission is weak because of the effects of turbulence in the arms.

The average equipartition strength of the total magnetic field for a sample of 74 galaxies is $\langle B_{\text{tot}} \rangle \sim 9\mu G$ (Niklas, 1995). Within that sample there is a large range of magnetic field strengths, from the very high $\approx 30\mu G$ seen in the spiral arms of M51 (Fletcher et al., 2004), shown in Fig. 1.4, through moderate magnetic fields of $\approx 15\mu G$ seen in M83 and NGC6946, to the lower magnetic field strengths in radio-faint galaxies such as M31 and M33. In starburst galaxies, the magnetic fields are the highest recorded for galaxies with $\langle B_{\text{tot}} \rangle \approx 50\mu G$ for M82, and even reaching $\sim 100\mu G$ for NGC 7552 (Beck, 2004) and NGC 1097 (Beck et al., 2005). If energy losses due to escaping electrons are significant then these values are lower limits (Beck and Krause, 2005). Recently, a field strength of $84\mu G$ was detected for a distant galaxy at a redshift of 0.692 using the Zeeman effect for the HI line in absorption against a quasar (Wolfe et al., 2008). In comparison, the strength of the regular field in spiral galaxies is typically much lower ($1 - 5\mu G$).

Results from edge-on galaxies and their halos show that in a number of cases (Dumke et al., 1995) the magnetic field runs parallel to the galactic plane for the inner regions. However, in other examples the direction of the magnetic field changes with galactic longitude (Krause et al., 2006; Heesen et al., 2007). Beck (2007) argues that energy equipartition produces a magnetic field scale height of approximately quadruple the scale height of the synchrotron-emitting thick disk.

### 1.4 Overview of Rotation Measure Studies

Studying the polarisation properties of synchrotron emission reveals the orientation of the magnetic field but not its direction. As discussed in Section 1.2.3, Zeeman splitting measurements yield $B_\parallel$ which will give the direction of the magnetic field (Troland and Heiles, 1986), but the long integration time, coupled with need for relatively high gas densities to enable detection, make this technique often impractical.

An alternative approach to studying magnetic fields is to use Faraday rotation. When a linearly polarised radio wave travels through a magnetised plasma, the intrinsic angle of
the polarised radiation, $\theta_0$, is rotated. The resultant orientation of the radiation is given by:

$$\theta = \theta_0 + \frac{e^3}{2\pi m^2 c^4} \lambda^2 \int_L n_e B_{\parallel} \, dl,$$

(1.9)

where $n_e(l)$ is the thermal electron density (in cm$^{-3}$), $B_{\parallel}(l)$ is the line-of-sight component of the coherent magnetic field (in $\mu$G) and $L$ is the distance from the source to the observer (in pc). The integral of the thermal electron density and the line-of-sight magnetic field component is called the rotation measure ($RM$) with units of rad m$^{-2}$. It is usually defined as:

$$RM = 0.81 \int_L n_e B_{\parallel} \, dl.$$

(1.10)

If $RM > 0$ then the magnetic field is directed towards us and if $RM < 0$ then the magnetic field is directed away from us. Faraday rotation is more widely used than Zeeman splitting because it can trace the magnetic fields of regions, such as the Galactic halo or the intergalactic medium (IGM) that are either too diffuse or too distant to be detected with the latter technique. The only requirement is a bright polarised source (such as a quasar) to provide the background radiation. Also, as Faraday rotation is most easily detectable at radio frequencies, it is not limited by interstellar extinction.
The first analysis of Faraday rotation of extragalactic sources was completed by Cooper and Price (1962). They determined that the phenomenon had a strong dependence on Galactic latitude which was confirmed in the following years by Gardner and Whiteoak (1966) and Seielstad et al. (1964). Since then many researchers have used Faraday rotation to probe the magneto-ionic medium of the Galaxy. Morris and Berge (1964) and Berge and Seielstad (1967) discovered a dependence on Galactic longitude as well. Other analyses throughout the 1970s, e.g., (Wright, 1973; Haves, 1975; Vallée and Kronberg, 1975) showed more details of the influence of the Galactic ISM.

With the discovery of pulsars (Hewish et al., 1968), another technique for determining the magneto-ionic properties of the Galaxy was discovered. Pulsars are ideal probes of the ISM, as they are often highly linearly polarised and the dispersion of the pulse as a function of frequency, known as the dispersion measure ($DM$), can be used:

$$DM = \int_L^0 n_e dl,$$

where $L$ is the distance to the pulsar. With the $RM$, the local average magnetic field can be determined:

$$\langle B_\parallel \rangle = \frac{\int_L^0 n_e B_\parallel dl}{\int_L^0 n_e dl} = 1.232 \frac{RM}{DM}.$$  

Calculating the magnetic field in this manner was first proposed by Smith (1968), but the author was unsuccessful in obtaining a magnetic field measurement. The first successful magnetic field measurement calculated in this manner was by Ekers et al. (1969) who measured the Galactic magnetic field using the Vela pulsar. A systematic analysis of pulsar RMs by Manchester (1972, 1974) was undertaken to determine the large scale Galactic magnetic field. Manchester concluded that the local field is uniform and directed towards Galactic longitude $l \sim 90^\circ$. Thomson and Nelson (1980) used pulsars to confirm that the local magnetic field was oriented in a clockwise direction, but also reported a field reversal near the Carina-Sagittarius arm. This field reversal in the Carina-Sagittarius arm was corroborated by Simard-Normandin et al. (1981) with their extensive RM catalogue of 555 extragalactic sources.

This large scale field reversal has generated much speculation as to the overall structure of the magnetic field in the Galaxy, which is most commonly modeled using symmetry arguments (Widrow, 2002). The two most common configurations expected are the axi-symmetric field configuration and the bi-symmetric magnetic field configuration. Fig 1.5 shows the vertical structure of the two different magnetic field modes, relevant in both the disc and the halo while Fig. 1.6 shows the azimuthal symmetries seen in the two magnetic field configurations. In addition to these models, which show spiral structure, an alternative distribution has been suggested by Rand and Kulkarni (1989), Rand and Lyne (1994) and Vallée (2005) in which the magnetic field lines do not align with the optical arms but form a series of concentric rings.

A large pulsar catalogue was published by Hamilton and Lyne (1987). Lyne and Smith (1989) used this catalogue to study the large scale Galactic magnetic field and
Figure 1.5  This figure shows the two most simple magnetic field configurations for galactic halos. In both panels there are two different magnetic field components, one vertical and the other horizontal. The symbols $\times$ and $\bullet$, denote oppositely directed magnetic fields, into and out of the page, respectively. In the left panel we have a quadrupolar field, which occurs when the horizontal field components are preserved above and below the Galactic plane, while the vertical components are in opposite directions. This field configuration has even parity. In the right panel there is a bipolar field, which has odd parity. In this case, the horizontal field reverses across the plane, while the vertical component maintains its direction. This figure is from Widrow (2002).

confirmed the reversal seen by Thomson and Nelson (1980). Subsequently, Rand and Lyne (1994) determined that the Crux-Scutum arm had a clockwise field. Following the release of the NE2001 electron density model of Cordes and Lazio (2002), which found that the Galactic disk was 33% more dense than previously expected, and thus that pulsars were 33% nearer than previously thought, a reanalysis of the pulsar data and the effect on the Galactic magnetic field was undertaken. Weisberg et al. (2004) found evidence for a number of field reversals. Han et al. (1999); Han and Wielebinski (2002) also discovered evidence for a counter-clockwise field in the Norma arm. Han et al. (2006) used their own measurements and an updated pulsar catalogue\(^2\) (Manchester et al., 1996) with 223 pulsars to investigate the large scale magnetic field structure in the Galaxy. The spiral arms themselves (Norma, Crux-Scutum, Carina-Sagittarius and Perseus) all have counter-clockwise fields when viewed from the north Galactic pole according to the authors. However, the inter-arm regions, have coherent clockwise fields leading Han et al. (2006) to propose a bi-symmetric model with reversals in arm-interarm regions (Fig. 1.6c).

Studies of the RM distribution across our Galaxy using extragalactic sources were first made by Gardner and Davies (1966) and subsequently Gardner et al. (1969). Simard-Normandin and Kronberg (1980) were the first to identify several large features such as “Region A” and they also showed that the RMs at high Galactic latitude were well-

\(^2\)http://www.atnf.csiro.au/research/pulsar/psrcat
1.4. Overview of Rotation Measure Studies

Different Magnetic Field Models

Concentric rings  Axi-symmetric spiral (ASS)  Bi-symmetric spiral (BSS)

Figure 1.6 This figure shows three large scale magnetic field models that have been suggested for the Milky Way. The first is a concentric ring model, in which the magnetic fields are aligned in rings rather than along the spiral arms. The second diagram shows an axi-symmetric field model for a face-on galaxy, while the third diagram is a bi-symmetric field model. Men et al. (2008) have reported that none are an ideal fit when all the observational evidence is considered. This figure is courtesy of Han (2004).

correlated with the large scale magnetic field. Other groups around the world have used combined extragalactic and pulsar sources to determine the large scale Galactic magnetic fields, including Brown et al. (2007) and Han (2008). The rationale for this approach is that increased extragalactic RM sampling from new surveys can provide more sightlines through the Galaxy than the relatively sparsely sampled pulsar RMs, while the pulsars give a better 3-D picture. However, local features, such as HII regions (Mitra et al., 2003) can cause significant changes to measured RMs. These anomalous RMs make it difficult to probe the global properties of the ISM. Also, the distance measurements of pulsars are based on models, such as that by Cordes and Lazio (2002) which are prone to modification. For example, Vallée (2008) has suggested an update to the NE2001 model with a new value for the Sun-Galactic-Centre distance, $R_{\odot} = 7.6$ kpc (Eisenhauer et al., 2005). When using extragalactic sources, there is a problem with calculating the integral $\int n_e B_{\parallel} dl$. In order to correctly ascertain the magnetic field strength a full model of the electron density, including all the cells in the IGM, needs to be constructed.

Brown and Taylor (2001); Brown (2002); Brown et al. (2003); Brown et al. (2007) used extragalactic sources to determine the large scale magnetic fields of the Galaxy, and in a recent paper (Brown et al., 2007) dispute the claims of (Han et al., 2006) that “unexpectedly positive” RMs in part of the Carina-Sagittarius arm may not be caused by HII regions along the line-of-sight. Brown et al. (2007) concluded that the best fit to their observational data was a clockwise field everywhere except for two counter-clockwise rings, one in the Crux-Scutum arm, and the other in the region of the Galaxy known as
the “molecular ring”, as shown in Fig. 1.7.

A small number of field reversals for the Galaxy is similar to the magnetic field structures seen in other galaxies (Beck, 2007). M81 has a bi-symmetric structure, similar to that proposed by Han et al. (2006) for our own Galaxy, but only two large scale reversals (Krause et al., 1989). M51 and NGC 4414 have only a single reversal if the overall structure of the magnetic field were viewed from inside the galactic disk (Shukurov, 2005). No other multiple field reversals have been detected in external galaxies. Beck (2007) suggests the inconsistencies result from the differing resolutions of the data. With external galaxies, averaging occurs over large sections of the galaxy, whereas in the Galaxy structures are measured to sub-pc level. As we discuss in Chapters 4 and 5, the Galactic magnetic field is turbulent on very small scales.

The concentric ring model for the Galactic magnetic field configuration has been championed by Vallée. Using pulsar data Vallée (2008) did a statistical analysis of the axi-symmetric field of Brown et al. (2007) and the bi-symmetric field configuration of (Han et al., 2006), as well as his own concentric ring model. Based on the statistics from this paper, the bi-symmetric model is not a good fit for the Galactic magnetic field. However, the axi-symmetric field and the concentric ring model are reasonable fits to the current data. Vallée (2008) argues it may be difficult to differentiate between a spiral and ring model at the present time.

More recently, Men et al. (2008) used observational values of $⟨B_∥⟩$ to derive the best fit parameters for each of the three models. They considered which model was consistent with pulsar data. Their work determined that while no model was a good fit for observations, the axi- and bi-symmetric fields provided better results than the ring model but that the actual large scale field was more complex than the current models allow. It is possibly a combination of both configurations.

It is clear, as has been noted by many authors, for example, Reich (2006) and Gaensler (2007), that more RMs are needed to produce an accurate model of the Galactic magnetic field. Chapter 6 provides the largest RM catalogue to date containing RMs from both extragalactic and pulsar sources. However, this is a small sample compared to the expected 200 million RM sources (Gaensler et al., 2004; Beck and Gaensler, 2004) that will be detected with the next generation radio telescope, the SKA.

### 1.5 Outline and Goals of Thesis

It is evident that magnetic fields are fundamental to our understanding of the universe. While detailed models and simulations are being developed (Mac Low, 2005; de Avillez and Breitschwerdt, 2006), observations are vitally important to validate and provide constraints for these models. Observing magnetic fields across all wavelengths can provide different information but to date, the observations at radio wavelengths have been the most successful in estimating the structure of the magnetic fields both in this Galaxy and in external sources.

The research described in this thesis examines the phenomenon of polarisation at ra-
Figure 1.7 This figure shows a possible model for the large scale magnetic field of the Southern Galactic Plane based on RMs of 149 extragalactic sources and 120 pulsars. The evidence presented by Brown et al. (2007) suggests a predominantly clockwise field with at least one, possibly two, localised reversals. The open arrow head in the Norma arm indicates that this field is not well constrained. This figure is from Brown et al. (2007).
dio frequencies as a probe of magnetic fields and their influence on the ISM. Chapter 2 is an introduction to polarisation. We discuss the polarisation capabilities of the Australia Telescope Compact Array (ATCA) and the issues faced in producing polarised images and how to interpret them. Chapter 3 briefly describes the theory of depolarisation and how this effect both hinders and helps our polarisation studies, including a particular depolarisation feature, depolarisation canals. Chapter 4 presents data taken with the ATCA and provides the first observational test of a diagnostic hypothesis proposed by Fletcher and Shukurov (2006); Fletcher and Shukurov (2007) that the type of depolarisation canal seen in polarised intensity images can be inferred from studying only the Stokes parameters. Observational evidence from this thesis was used to develop a more robust method to identify depolarisation canals. Chapter 5 continues the investigation of depolarisation canals, using the ATCA data from Chapter 4 and results from Wolleben taken from the DRAO all-sky polarisation survey. These observational data are used to introduce a completely new way of looking at depolarisation canals. We believe that this method will help understand ISM turbulence and make depolarisation canals a more generic tool for studying the ISM. Chapter 6 looks at a different polarisation phenomenon, rotation measures. We utilise data taken over more than 40 years, from 1963 – 2007, to produce a comprehensive catalogue of RMs across the entire sky. A preliminary interpretation of the Galactic magnetic field structure and a brief analysis of RM/redshift correlations have been made. Chapter 7 summarises the work conducted in this thesis and looks to the future.

The significance of this work is to enhance understanding of the phenomenon of polarisation in an astrophysical context. We have developed techniques to interpret the complex diffuse polarised emission observed, which in turn allows a better understanding of the Galactic magnetic field on small scales and the turbulent nature of the ISM. The catalogue of RMs also helps to answer questions regarding the presence of magnetic field reversals in our Galaxy. Field reversals have been predicted (Shukurov, 2005) but are rare in other galaxies (Beck, 2005). Recent work by Brown et al. (2007), suggests that there is at least one field reversal between the local Orion arm and the Sagittarius arm, but as Reich (2006) comments, larger RM data sets are needed to resolve this issue. Our new large dataset can partially answer the question. On a more global scale, we need to understand the complexities of our own Galaxy structures and its magnetic field so that we can refine the model required for precision cosmic microwave background (CMB) studies.
Chapter 2

A Primer on Polarisation

2.1 Introduction

In this chapter we shall describe techniques to study the phenomenon of polarisation. The polarisation of electromagnetic radiation is fundamental to many physical processes including: how bees view the world (von Frisch, 1948), why rainbows exist (see (Graham, 1975) for a review) and the explanations for many astrophysical phenomena. A radio interferometric telescope is a valuable instrument for obtaining high quality and spatial resolution polarisation information. However, the procedures to calibrate the data are complex and need to be undertaken accurately to produce meaningful results. The polarisation calibration of a radio interferometer is described in this chapter, with the Australia Telescope Compact Array as a particular case, leading on to descriptions of the generation and interpretation of polarisation images. There have been papers which discuss polarisation in detail, but there are many traps for novices. The aim of this chapter is to present the basic ideas behind polarisation image processing and to highlight some of the pitfalls to be avoided.

2.2 The Basics of Polarisation

Electromagnetic waves can be characterised by four properties: intensity, frequency, direction of propagation and polarisation state. Polarisation was originally described in 1808 by Malus when he observed the reflection of the setting sun in a window through a crystal of Icelandic spar. And it was polarisation that helped Michael Faraday establish a connection between light and electromagnetism in the 1840's. Faraday (1844) showed that a piece of isotropic glass becomes birefringent when threaded by a strong magnetic field. The mathematical basis for the hypothesis that light is simply transverse electromagnetic radiation was completed by Maxwell.

Fig. 2.1 shows why polarisation is considered an important characteristic of an electromagnetic wave. The oscillating electric and magnetic field vectors will have a particular orientation at a given instant in time (Fig. 2.1(a)). Fig. 2.1(b) shows how the polarisation state of a wave changes its orientation over time. If the wave oscillates in a single
Figure 2.1 This figure shows an electromagnetic wave (a) and how its polarisation changes as it propagates along the z-axis (b). At position 1, the electric vector is pointing in the x-direction, but at point 2, some time in the future, the electric vector is pointing in the y-direction. One complete revolution has been completed by point 4, and points 5 – 8 repeat the cycle. The wave is circularly polarised as there are equal amplitude components in both the x and y planes. This figure is reproduced from http://astronomyonline.org

plane, the wave is said to be linearly polarised (see Fig. 2.2(b)), while a wave composed of two plane waves of equal amplitude but differing in phase by 90° is circularly polarised (Fig. 2.2(a)). If two plane waves have differing amplitudes or a phase difference that is not 90°, then the resultant wave is elliptically polarised (Fig. 2.2(c)). There are many ways of describing the polarisation state of an electromagnetic wave. If we begin with a quasi-monochromatic wave propagating in the z-direction, it can be resolved into three Cartesian components (x, y, z):

\[
\begin{align*}
E_x &= a_1(t) \exp i(\phi_1(t) - 2\pi \nu t), \\
E_y &= a_2(t) \exp i(\phi_2(t) - 2\pi \nu t), \\
E_z &= 0,
\end{align*}
\]

(2.1)

where \( E \) is the magnitude of the electric field vector, \( a_1, a_2 \) are the amplitudes and \( \phi_1, \phi_2 \) are the phases of the component waves. The polarisation states of the wave are described by the Stokes parameters (derived in Appendix A of this thesis) (Born and Wolf, 1959):
2.2. The Basics of Polarisation

2.2.1 The Poincaré Sphere

The Poincaré sphere was formulated by Poincaré (1892) with the aim to show simply the polarisation state of electromagnetic waves. A Poincaré sphere is shown in Fig. 2.3. At the two poles the radiation is completely circularly polarised. In the representation shown, the LCP pole is at the top of the sphere (in the direction of $V > 0$), while the RCP pole is at the bottom of the sphere (in the direction of $V < 0$). Complete linear polarisation occurs along the equator of the sphere, and other points on the sphere correspond to elliptical polarisation of varying degree. The longitude ($2\psi$) represents the tilt angle of the radiation while the latitude ($2\chi$) represents the ellipticity. The angles are doubled here because of the dipole nature of polarised radiation. A change of $180^\circ$ will produce a
Chapter 2. A Primer on Polarisation

Figure 2.3 A representation of the Poincaré Sphere. The polarisation at each cardinal point is labelled. Along the equator the polarisation is fully linearly polarised, while at the poles it is fully circularly polarised.

vector that is indistinguishable from the original spin-2 vector. By doubling the angles, the representation of the polarised radiation on the Poincaré sphere is easier to interpret. The radius of the sphere is given by \( S_0 \), the total flux of the electromagnetic wave. The three Stokes parameters are the Cartesian axes shown in the sphere. Fig. 2.4 shows the relationship between the Poincaré sphere and the Stokes parameters. The expressions are as follows:

\[
S_0 = I = E_a^2 + E_b^2, \\
S_1 = Q = S_0 \cos 2\chi \cos 2\psi, \\
S_2 = U = S_0 \cos 2\chi \sin 2\psi, \\
S_3 = V = S_0 \sin 2\chi. 
\] (2.3)

Representing the radius as the total flux of the radiation was first proposed by Pancharatnam, (Radhakrishnan, 1990), allowing both the degree of total polarisation and the components of \( Q, U \) and \( V \) to be described by the angle and length of the vector on the sphere. For a monochromatic wave which is 100% polarised, the tip of the vector on the Poincaré sphere would reach the surface of the sphere at some angle. A completely unpolarised vector is shown as a point at the origin with a value designated as the “zero component”. For partially polarised radiation, the length of vector will be less than \( S_0 \) in a particular direction, with the zero component at the centre of the sphere comprising the unpolarised part of the vector (see Fig. 2.4). An explanation of the zero component is given in Radhakrishnan (1990). The Poincaré sphere can be useful in visualising polarisa-
2.2. The Basics of Polarisation

Figure 2.4 The representation of different levels of polarisation. On the left we have the normal Poincare sphere, labelled with the Stokes parameters, $Q$, $U$ & $V$. The radius of this Poincaré sphere is the total intensity, $I$. The polarised vector '$S_0$' is an example of a polarised vector. On the right three different scenarios are drawn. The top shows an unpolarised wave, drawn as a point at the origin with no components in $Q$, $U$ or $V$. The middle picture shows partial polarisation, where the tip of the vector does not reach the edge of the sphere. The final case is that of a fully polarised signal. The polarised vector touches the surface of the sphere, since $I^2 = Q^2 + U^2 + V^2$ for a fully polarised monochromatic wave. Figure adapted from Radhakrishnan (1990).

...tion. For example, if the radiation were unpolarised, then integration over a long period of time, would result in a uniform sphere, as there is no preferred orientation. However, for highly polarised radiation, the time-averaged integration would produce a narrow beam in a particular direction, while for partial polarisation, the area covered would be a broader region on the sphere.

2.2.2 Interferometric Polarisation

Radio antennas are inherently sensitive to a preferred plane of polarisation which means polarisation has always been a consideration when building radio telescopes. To measure the total intensity of the radiation, two receiving elements with orthogonal polarisation planes are required. Radio astronomers needed to develop a specific polarisation protocol as optical polarisation theory is not directly applicable.
The first to present a detailed discussion of polarisation using a radio interferometer was Morris et al. (1964). For an ideal two element interferometer (each with a single receiver) they derived the output correlation coefficients from the input Stokes visibilities. Subsequent work (Weiler, 1973; Thompson et al., 2001) included various instrumental uncertainties. A comprehensive theoretical framework was published in a series of papers by Hamaker et al. (1996) and Sault et al. (1996). The first paper (Hamaker et al., 1996) presented the mathematical foundation of radio polarimetry with an interferometer, while the second paper showed how to apply the mathematical theory to real-world interferometers. The most fundamental concepts relating to interferometric polarimetry are presented in this section. For more detailed and rigorous mathematical proofs, see Hamaker et al. (1996) and Thompson et al. (2001). Only a linearly-polarised interferometer is described because this is the system used in the Australia Telescope Compact Array (ATCA). Hamaker et al. (1996) consider the general case of a polarised system.

In an interferometer the two orthogonal receiving elements are each sensitive to one direction of radiation. For example, the $E_x$ receiver may be sensitive to radiation polarised in the $x$-direction, and the $E_y$ receiver is sensitive to radiation polarised in the $y$-direction. In this system where we assume perfect feeds with no gains, the measured visibilities can be written in vector format as:

$$v = \begin{pmatrix} v_{xx} \\ v_{xy} \\ v_{yx} \\ v_{yy} \end{pmatrix},$$  \hspace{1cm} (2.4)

where $v_{ab}$ is the cross correlation between a voltage from antenna ‘a’ and a voltage from antenna ‘b’. Equation 2.4 is often known as the “coherency vector” ($Z_k(a,b)$). The spatial coherence function is given by the product of the radiation field $E_k$ from antenna ‘a’ and the complex conjugate radiation field $E_k^*$ from antenna ‘b’, i.e.:

$$Z_k(a,b) = \langle E_k(a) \rangle \langle E_k^*(b) \rangle.$$

We can also express the vector products from Eq. 2.4 in terms of the Stokes visibilities by using Eq. 2.2:

$$Z_{xx}(u,v) = I(u,v) + Q(u,v),$$
$$Z_{yy}(u,v) = I(u,v) - Q(u,v),$$
$$Z_{xy}(u,v) = U(u,v) + iV(u,v),$$
$$Z_{yx}(u,v) = U(u,v) - iV(u,v),$$

where $I/Q/U/V(u,v)$ are the visibilities, which are converted into brightness distributions by using Fourier transforms. The converted Stokes parameters are related to the correlation products by rearranging Eq. 2.6:

$$I = \frac{(Z_{xx} + Z_{yy})}{2},$$
2.2. The Basics of Polarisation

\[ Q = \frac{(Z_{xx} - Z_{yy})}{2}, \]
\[ U = \frac{(Z_{xy} + Z_{yx})}{2}, \]
\[ V = \frac{(Z_{xy} - Z_{yx})}{2}. \] (2.7)

The above equation (Eq. 2.7) is for an ideal system, and for “real-world” instruments the process of obtaining the Stokes parameters is more complicated. The inputs from the \( E_x \) and \( E_y \) feeds have additional terms because the receiving elements are not perfectly polarised, and can be represented thus:

\[
\begin{pmatrix}
E'_x \\
E'_y
\end{pmatrix} = \begin{pmatrix}
a & b \\
c & d
\end{pmatrix} \begin{pmatrix}
E_x \\
E_y
\end{pmatrix}. \] (2.8)

The \( 2 \times 2 \) matrix is commonly known as a “Jones matrix”, invented by Jones (1941). With interferometric polarisation, the Jones matrix is composed of a number of different components, which can be represented in the following manner:

\[ J_{\text{overall}} = J_1 J_2 J_3. \] (2.9)

Polarised signals can be corrupted in a number of ways in their passage from the source to the receiver. Modification of the signal can be from propagation through the ionosphere, or instrumental effects from the antenna elements or other parts of the signal pathway, such as the correlator. The polarised receiving elements (or feeds) do not have perfect polarisation purity, which means that some of the \( E_x \) field will be coupled into the “\( y \)” feed. This is particularly critical when the feeds rotate with respect to the sky, as they do in an alt-az\(^1\) antenna such as the ATCA. Through careful design, the feeds can be made to respond linearly to the radiation, and so the excess terms, known as leakages, can be simplified to:

\[ E_X = E_x + d_X E_y, \]
\[ E_Y = E_y - d_Y E_x, \] (2.10)

where \( d_X, d_Y \) are the complex leakages. The leakage terms can also be represented in terms of a Jones matrix:

\[ J_{\text{leakage}} = \begin{pmatrix}
1 & d_X \\
-d_Y & 1
\end{pmatrix}. \] (2.11)

The real and imaginary parts of the leakages have a physical interpretation. The real part is caused by misalignment between the two feeds, i.e. any deviation from 90° between the two feeds will produce a leakage signal. The imaginary part is called the ellipticity. It is impossible to machine a perfect feed that is only sensitive to a single plane of polarisation. If the feed has any sensitivity to the orthogonal polarised radiation, then the signal received is slightly elliptically polarised.

\(^1\)An alt-az or elevation-over-azimuth mount is one type of mount for a reflector antenna. For more details see Thompson et al. (2001) pp. 94 – 96
Chapter 2. A Primer on Polarisation

The other terms contributing to the component Jones matrices are the gain terms:

\[ J_{\text{gain}} = \begin{pmatrix} g_x & 0 \\ 0 & g_y \end{pmatrix}, \]

(2.12)

where \( g_x, g_y \) are the complex gain factors for the two orthogonally polarised signals. The gains can represent both instrumental effects in the electronics, but also extraneous signal coming from objects other than the source of interest, for example, from the atmosphere.

Another component Jones matrix represents the feed rotation contribution, which can result when the feed parallactic angle for an alt-az mount changes as the telescope tracks a source across the sky, or can be caused by Faraday rotation (explained in Section 1.4 and examined in detail in later chapters). For orthogonal-linear feeds, the rotation matrix is given by:

\[ J_{\text{rotation}} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}, \]

(2.13)

where \( \phi \) is the rotation angle between the feed and the source. The measured field (\( E_K \)) can be related to the actual field in the following manner:

\[ E_X = E_x \cos \phi + E_y \sin \phi, \]
\[ E_Y = -E_x \sin \phi + E_y \cos \phi. \]

(2.14)

In general, the Jones matrices vary for different antennas and as a function of frequency and time. When the system is calibrated all components must be included.

2.2.3 Calibration of Interferometric Polarisation

Calibration of interferometric polarisation is complicated. For every correlation, for each antenna and both feeds, the gains, leakages, and rotation need to be estimated. Initially, it was believed that it was not possible to accurately calibrate linearly polarised feeds, but a suggestion by Komesaroff (1985) to inject a known noise signal to provide phase coherent noise which is stable over long periods produced a solution (see Fig. 2.5). By measuring the total power outputs from the noise source and comparing the results with the gains, phases and leakages of a known calibrator the gains for the target source can be obtained and an accurate measurement of the Stokes parameters results.

A simple approximation can be made for interferometric calibration of a weakly polarised source and nearly perfect feeds. In this case, most of the leakage terms can be ignored and the cross-correlations in a two-antenna baseline \( (A,B) \) with linearly polarised feeds become the following:

\[ Z_{xx} = \frac{1}{2} g_{Ax} g_{Bx}^* (I + Q \cos 2\chi + U \sin 2\chi), \]
2.2. The Basics of Polarisation

Figure 2.5 The maximum number of independent outputs from a radio antenna is given by $\sqrt{\Delta \nu \Delta t}$. In the Rayleigh-Jeans regime, the number of states in a system is less than the number of photons, so from Bose-Einstein statistics we know that large numbers of photons are in the same state. Therefore, splitting the noise signal will result in identical copies being directed into each feed of the antenna, allowing us to calibrate linearly polarised radiation.

$$
Z_{yy} = \frac{1}{2} g_{Ay} g_{By}^* (I - Q \cos 2\chi - U \sin 2\chi),
$$

$$
Z_{xy} = \frac{1}{2} g_{Ax} g_{By}^* (I(d_{Ax} - d_{By}^*) - Q \sin 2\chi + U \cos 2\chi + iV),
$$

$$
Z_{yx} = \frac{1}{2} g_{Ay} g_{Bx}^* (I(d_{Bx}^* - d_{Ay}) - Q \sin 2\chi + U \cos 2\chi - iV). \quad (2.15)
$$

However, the leakages need to be considered when solving for the cross-correlation coefficients for a strongly polarised source. According to Sault et al. (1991) the leakage product terms $(d_{x,y})$ can be ignored as they are usually small. The measured correlations in the strongly polarised case are:

$$
Z_{xx} = \frac{1}{2} g_{Ax} g_{Bx}^* [I + Q(\cos 2\chi - (d_{Ax} + d_{Bx}^*) \sin 2\chi)
+ U(\sin 2\chi + (d_{Ax} + d_{Bx}^*) \cos 2\chi) - iV(d_{Ax} - d_{Bx}^*)];
$$

$$
Z_{yy} = \frac{1}{2} g_{Ay} g_{By}^* [I - Q(\cos 2\chi - (d_{Ay} + d_{By}^*) \sin 2\chi)
- U(\sin 2\chi - (d_{Ay} + d_{By}^*) \cos 2\chi) + iV(d_{Ay} - d_{By}^*)];
$$

$$
Z_{xy} = \frac{1}{2} g_{Ax} g_{By}^* [I(d_{Ax} + d_{By}) - Q(\sin 2\chi + (d_{Ax} - d_{By}^*) \cos 2\chi)
+ U(\cos 2\chi - (d_{Ax} + d_{By}^*) \sin 2\chi) + iV],
$$
\[ Z_{yx} = \frac{1}{2} g_A y^* g_B x^* [I(d_{Ay} + d_{Bx}^*) - Q(\sin 2\chi - (d_{Ay} - d_{Bx}^*) \cos 2\chi) \]
\[ + U(\cos 2\chi + (d_{Ay} + d_{Bx}^*) \sin 2\chi) - iV]. \quad (2.16) \]

From the above set of complex equations, it is apparent that a solution cannot be obtained using only two antennas. To overcome this problem, multiple baselines need to be observed. Alternatively, these cross-correlation products can be found by observing a calibrator with known polarisation properties which is used to solve for the instrumental gains, leakages and parallactic rotations. The equations cannot be solved for the absolute level of the leakages as those terms only ever appear together, so any arbitrary offset and its complex conjugate will not affect the cross-correlation equations from Eq. 2.16. Thus we have two inseparable degrees of freedom. There is another degree of freedom that also cannot be solved for and that is the phase relationship between the \( x \) and \( y \) gains. As long as at least three baselines are used (for triple product calculations) then there will always be three degrees of freedom that are not able to be numerically determined. However, these unsolvable products do not prevent correct calibration, provided that reasonable constraints are added to the solution.

Before these “reasonable constraints” can be included, a reality check for a feasible astrophysical solution is required. In the following section the work of Sault et al. (1991) is summarised as well as the polarisation papers of Hamaker et al. (1996) and Sault et al. (1996).

The measured Stokes parameters are related to the true Stokes parameters in the following way:

\[
\begin{pmatrix}
I' \\
Q' \\
U' \\
V'
\end{pmatrix} = \begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix} + \begin{pmatrix}
\Delta I \\
\Delta Q \\
\Delta U \\
\Delta V
\end{pmatrix},
\]

(2.17)

where the \( I' \), etc. parameters are the measured Stokes parameters, \( I \) etc. are the true source Stokes parameters and \( \Delta I \) etc are the measured uncertainties. In order to determine the error terms we start with the gains and their associated errors, assuming there is an ideal instrument with gains \( g_{A,x} \), \( g_{A,y} \) equal to 1, then:

\[
g'_{A,x} = 1 + \Delta g_{A,x}, \\
g'_{A,y} = 1 + \Delta g_{A,y},
\]

(2.18)

where \( \Delta g \) are the uncertainties in the gains. The nominal Stokes parameters will be related to the measured values in the following way, based on Eq. 2.16 and following Sault et al. (1991):

\[
I' = I + 1/2[I\epsilon^{++} - Q(\epsilon^{--} \sin 2\chi - \zeta^{++} \cos 2\chi) + U(\epsilon^{+-} \cos 2\chi + \zeta^{-+} \sin 2\chi) + iV\zeta^{--}],
\]
\[ Q' = Q + \frac{1}{2}[-I(\epsilon^+ \sin 2\chi - \zeta^{++} \cos 2\chi) + Q\epsilon^{++} - U\zeta^{-+} - iV(\epsilon^- \cos 2\chi + \zeta^{+-} \sin 2\chi)], \]

\[ U' = U + \frac{1}{2}[I(\epsilon^+ \cos 2\chi - \zeta^{++} \sin 2\chi) + Q\zeta^{-+} - U\epsilon^{++} - iV(\epsilon^- \sin 2\chi - \zeta^{+-} \cos 2\chi)], \]

\[ iV' = iV + \frac{1}{2}[-I\zeta^{-+} - Q(\epsilon^- \cos 2\chi + \zeta^{+-} \sin 2\chi) - U(\epsilon^- \sin 2\chi - \zeta^{+-} \cos 2\chi) + iV\epsilon^{++}], \]

(2.19)

where (from Eqs. 2.16, 2.17 & 2.18) (Sault et al., 1991) and (Sault et al., 1996):

\[ \epsilon^{++} = (\Delta g_{A,x} + \Delta g_{A,y}) + (\Delta g^*_{B,x} + \Delta g^*_{B,y}), \]

\[ \epsilon^{--} = (\Delta g_{A,x} - \Delta g_{A,y}) - (\Delta g^*_{B,x} - \Delta g^*_{B,y}), \]

\[ \epsilon^{+-} = (\Delta g_{A,x} - \Delta g_{A,y}) + (\Delta g^*_{B,x} - \Delta g^*_{B,y}), \]

\[ \zeta^{++} = (d_{A,x} + d_{A,y}) + (d^*_{B,x} + d^*_{B,y}), \]

\[ \zeta^{--} = (d_{A,x} - d_{A,y}) - (d^*_{B,x} - d^*_{B,y}), \]

\[ \zeta^{+-} = (d_{A,x} - d_{A,y}) + (d^*_{B,x} - d^*_{B,y}), \]

\[ \zeta^{--+} = (d_{A,x} + d_{A,y}) - (d^*_{B,x} + d^*_{B,y}). \]

(2.20)

All the \( \epsilon \) terms relate to the gains of the system, while all the \( \zeta \) terms relate to the leakages. As mentioned before (Section 2.2.2), these error terms are most likely to be complex and finite, implying that the deduced Stokes visibilities will also be complex and finite, and most probably different for each baseline. Each of the above parameters can affect the determination of the true gains and phases for the system. Table 2.1 (Sault et al., 1996) summarises how incorrect error calculation will corrupt the calibration of the interferometric data.
Table 2.1: The relationship between undetermined calibrator and instrumental parameters and the seven degrees of freedom in the calibration solutions, from Sault et al. (1991) and Sault et al. (1996)

<table>
<thead>
<tr>
<th>Error Term</th>
<th>Source of Error</th>
<th>Consequence of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon^{++}$</td>
<td>Inaccurate calibrator flux</td>
<td>Incorrect flux scale</td>
</tr>
<tr>
<td>$\epsilon^{--}$</td>
<td>Incorrect phase determination</td>
<td>Leakage between $Q$ &amp; $V$</td>
</tr>
<tr>
<td>$\epsilon^{+-}$</td>
<td>Inaccurate $Q$ measurement</td>
<td>Leakage between $I$ &amp; $Q$</td>
</tr>
<tr>
<td>$\zeta^{+-}$</td>
<td>Inaccurate $U$ measurement</td>
<td>Leakage between $I$ &amp; $U$</td>
</tr>
<tr>
<td>$\zeta^{++}$</td>
<td>Inaccurate $V$ measurement</td>
<td>Leakage between $I$ &amp; $V$</td>
</tr>
<tr>
<td>$\zeta^{--}$</td>
<td>Incorrect real leakage determination</td>
<td>Linear polarisation angle error</td>
</tr>
<tr>
<td>$\zeta^{+-}$</td>
<td>Incorrect imaginary leakage determination</td>
<td>Leakage between $U$ &amp; $V$</td>
</tr>
</tbody>
</table>

The parameters $\epsilon^{+-}, \zeta^{+-}, \zeta^{--}$ relate to the strength of the polarisation of the calibrator. If the values are wrong, then the polarisation of the target source will be in error by an equal and opposite amount. These three parameters can always be determined, even if the absolute polarisation of the calibrator is unknown. The remaining error term is $\epsilon^{++}$, and while the absolute flux cannot be determined if we do not know the properties of the calibrator, it is still possible to determine $\epsilon^{++}$ if we know the flux density of the target source. However, even knowing the polarisation properties of a calibrator still leaves three unconstrained terms $\epsilon^{--}, \zeta^{++}, \zeta^{--}$. A number of methods have been developed to overcome this limitation. The physical orientation of the feeds can be used to determine $\zeta^{++}$. The other two parameters relate to circular polarisation, which can usually be assumed to be negligible. Another approach, suggested by Sault et al. (1996) is to observe three different calibrators all of known polarisation properties with at least one with linear polarisation. Such a sample of sources would enable a unique calibration solution. However, the limitation of this approach is the requirement for a large number of calibrators distributed over the sky with very well known Stokes parameters. Given that many sources are variable on various time scales, it is usually considered to be an impractical task to continuously monitor a large set of calibrators to guarantee accurate solutions.

There is one other technique that can be used to determine the calibration parameters. It works with an alt-az mounted antenna with fixed feeds, that thus rotate with respect to the sky over a full 12-hour synthesis. The presumption is that the calibrator Stokes parameters vary with the parallactic angle in a predictable way and that the leakages and
2.3 The ATCA Polarisation System

The ATCA (Australia Telescope Compact Array) is a synthesis radio interferometer consisting of six 22-m diameter, alt-az, Cassegrain focus antennas, located at the Paul Wild Observatory, near Narrabri, New South Wales (Frater et al., 1992). Five of the six antennas can move between 37 defined stations along a 3-km east-west track and 5 stations along a 200-m northern spur which intersects the east-west track. The ATCA was built on a tangent line to the Earth, but the surveyors found they could not place the tangent at the centre of the array. So the tangent point for the Compact Array is approximately 3 kms west of Antenna 6, and the array was changed to minimise the construction costs. The array itself runs parallel to the equatorial plane, but for a longitude slightly different than the actual site (Kesteven, private communication). The sixth antenna is located a further 3-km to the west of the main track. This antenna cannot be moved and produces a maximum baseline of 6 km. The 6 antennas provide 15 baselines in a variety of configurations giving a range of resolutions and uv-coverage (Duncan et al., 2006). The ATCA operates as a national facility and is open to the astronomical community through a peer-review process.

Each ATCA antenna has a number of wide-band, compact corrugated feed horns placed on a rotating turret which can be changed remotely. Observations are made with the feeds on axis to minimize instrumental polarization errors. At the time of writing there were seven frequency bands available at the ATCA, which are: 20 cm (1.25 – 1.78 GHz), 13 cm (2.20 – 2.50 GHz), 6 cm (4.40 – 6.70 GHz), 3 cm (8.00 – 9.20 GHz), 12 mm (16 – 26 GHz), 7 mm (30 – 50 GHz) and 3 mm (85 – 105 GHz). Each feed horn is equipped with a pair of orthogonal linear probes and simultaneous observation through two intermediate frequency chains is possible. The ATCA User Guide (Duncan et al., 2006) provides details of the combinations of frequency and observing modes that are offered. Observations can currently be undertaken as narrow channel spectroscopy, or in high-sensitivity “continuum” mode, where 32 channels are used to cover a maximum of 128 MHz bandwidth. Of the 32 channels in continuum mode only 16 are independent, allowing the correct gridding of \((u, v)\)-plane data into the spatial frequency domain, which is known as multi-frequency synthesis. Splitting the continuum band into channels enables narrow-
band radio frequency interference (RFI) to separately be removed without corrupting the entire data sample. The ATCA has good sensitivity because of broad bandwidth, which compensates for a modest collecting area. Good $uv$-coverage requires the combination of several antenna configurations.

When the ATCA antennas are in an east-west configuration at any instant there is only 1D coverage in the $(u,v)$-plane. Complete azimuthal coverage comes from an observation of 12 hours, where the variation in each projected antenna baseline provided by the rotation of the Earth allows a two-dimensional visibility data set to be formed. Fig. 2.6 shows some examples of baseline ellipses. With the advent of the northern spur at the ATCA, good $(u,v)$-coverage can be achieved in less than 12 hours.

One of the engineering challenges for the ATCA was to choose between circularly polarised or linearly polarised feeds. At the time, the conventional wisdom was that interferometers could not be calibrated with linearly polarised feeds because of their parallactic angle dependence (discussed in the previous section), which can cause artifacts to appear in the maps. However, linearly polarised feeds have technical advantages over circularly polarised feeds because they can support wide-band systems and the polarisation purity remains high over a much broader frequency range. The decision to use linearly polarised feeds was taken when Komesaroff solved the parallactic angle problem. His solution was to use a noise diode to allow continuous measurement of the phase difference between the feeds (the $xy$-phase). The instrumental polarisation in the uncalibrated ATCA array is typically below 2–3% and with a good calibration solution applied (Section 2.2), the instrumental effects are reduced to approximately 0.1%. The off-axis polarisation error is slightly more. The 20 and 6 cm bands have the best polarisation characteristics while the 13 cm is not as good, due to an error in its feed horn design (Sault and Ehle, 1996). Full synthesises can reduce some errors as the off-axis response is smeared out.

One of the observational modes used in this thesis is mosaicing (Ekers and Rots, 1979), a technique to map several pointing centres in a 12-hr run, which are then combined into an image, often before deconvolution of the $uv$-data (Cornwell, 1988). Mosaicing produces large images with an improved and more uniform signal-to-noise ($S/N$) ratio over the entire field of view. The improved $S/N$ occurs because adjacent pointings are chosen so they are not independent. It can potentially be difficult to maintain polarisation purity across a wide field, but the turret rotation on the ATCA enables the polarisation characteristics of a source to be accurately reproduced over the full field of view. Mosaicing also has an advantage in polarisation imaging because it further reduces the off-axis polarisation response. For more information, see (Duncan et al., 2006) and Frater et al. (1992).

2.4 Deriving Polarisation Images

As shown in Section 2.2 the steps involved in calibrating an interferometer require a good estimation of the gains, leakages and parallactic angle to be made for every feed for each individual correlation. The aim of the calibration procedure is to apply corrections to mimic an ideal interferometer with gains of unity and zero leakages. Most polarisation observations from the ATCA are processed through the software program MIRIAD.
Figure 2.6 This figure shows data for a single frequency channel, taken over a 12 hr synthesis with five antennas of the ATCA, the 6 km antenna was not included in this plot. Only half of these points in the figure are physically measured by the telescope. The other half can be reconstructed by symmetry arguments, and allows us to plot the behaviour of the source over a full rotation of the Earth. The \((u, v)\)–plane is measured in \(k\lambda\), while the different ellipses correspond to different baselines.
(Sault et al., 1995), which will be described in the next section.

### 2.4.1 Data Importation and Pre-calibration, including Flagging

ATCA data are usually taken with an integration time of 10–30 seconds. In continuum mode all four correlation products ($Z_{xx}, Z_{yy}, Z_{xy}, Z_{yx}$) for each baseline pair are recorded as an RPFTS file (Norris, 1985). The data file must be converted into a MIRIAD compatible format, using the task ATLOD, before any processing can begin. ATLOD also corrects for the $xy$-phase difference measured by the noise diode, removes the poor quality edge channels and known RFI channels. Only 16 of the 32 channels observed are independent so another of ATLOD’s functions is to suppress non-independent channels, leaving 13 contiguous 8 MHz channels, centred on the selected central observing frequency. In spectral line mode, ATLOD has slightly different functionality, which is described in detail in the MIRIAD User’s Guide\(^2\).

The next task before calibration occurs is to split the observations into the separate observing frequencies. The task is UVSPLIT, which also by default, separates the data by source name. If an observation has been undertaken in mosaicing mode, then UVSPLIT collates the various pointing centres into the same source file.

**Flagging**

Flagging is the process whereby bad data are excluded from any further processing. Unusable data occur when the telescope is not tracking the source, and these are automatically excised. An example of bad data could be a brief burst of RFI. This is not removed automatically, and must be done after the observations have been completed. There are a number of processes to undertake flagging, including UVCLIP, TVCLIP, UVFLAG, TVFLAG and BLFLAG. The “clip” tasks automatically remove data that has an intensity above a certain level. This can be useful if the dataset being flagged contains intermittent RFI. However, it is unwise to universally clip the data, particularly when sources with complex structure are observed. Commonly, it is prudent, though more time consuming, to use one of the three main MIRIAD flagging routines, UVFLAG, TVFLAG and BLFLAG. In MIRIAD flagging does not delete the data. They are masked and can be recovered if a revision is needed. Flagging can be very time-consuming and observations at 20 and 13 cm are more prone to RFI corruption than shorter frequencies. However, a new flagging method, PIEFLAG (Middelberg, 2006) utilises two algorithms to filter out measurements that are affected by interference and has cut flagging time significantly.

### 2.4.2 Calibration

As shown in Section 2.2, calibration is one of the most important aspects of producing an image. Some observations only have two correlation products recorded ($Z_{xx}$ and $Z_{yy}$) and cannot be used to measure polarization. However, for this thesis we will focus on calibration relevant to polarisation observations. With all four polarisation products, it is

\(^2\)http://www.atnf.csiro.au/computing/software/miriad
possible to solve for the leakage terms, the gains and a bandpass function and calibration of Q and U. Figure 2.7 shows the usual calibration procedure for polarisation data taken from the Miriad User’s Guide.

The initial action is to determine the absolute flux density scale by using a short observation of the ATCA primary calibrator, PKS B1934-638. First, we solve for antenna gains and bandpass function using the Miriad task MFCAL, which assumes an unpolarised calibrator. However, with all four polarisation products recorded any polarised flux can be corrected later in the calibration procedure. MFCAL contains as reference the spectra of a number of well-known suitable flux calibrators, including PKS B1934-638.

The second step in the primary flux calibration is the determination of the antenna gains and the instrumental polarisation parameters (leakages). All of these parameters are complex quantities, but the gains are time-dependent while the leakages are assumed to be time-independent. The task GPCAL is used for this process, utilising the bandpass calculated in MFCAL and solving for any residual $xy$-phase term remaining from the ATLOD task. This completes the flux scale calibration.

After the primary calibration there are three degrees of freedom ($\zeta^{++}$, $\zeta^{--}$ and $\epsilon^{--}$) which represent the polarisation level of the calibrator. The task GPCAL assumes nominal alignment ($\zeta^{++}$) and ellipticity ($\zeta^{--}$) for the $x$ feed on the specified reference antenna. The final parameter to constrain is the $xy$-phase, $\epsilon^{--}$, which is set to zero on this reference antenna. Because the primary calibrator only sets the flux density scale, the constraint on these polarised quantities may be amended using the secondary calibration source.

The secondary calibrator is also known as the “phase” calibrator. In continuum centimetre observing, the phase calibrator is observed once every 30 – 45 minutes and its role is to provide a reference for any atmospheric or instrumental effects that might occur during the course of the observations. The calibrator is chosen to be located ideally within 10° of the target source and should be relatively strong (at least 0.5 Jy is desirable). At 20 cm it is also essential to make sure the calibrator is not confused and this usually requires a flux of $> 0.5 \text{ Jy}/\text{beam}$. A similar procedure as for the primary calibrator is followed. The bandpass function is determined by using MFCAL, and while the bandpass is known very accurately for the primary calibrator, the secondary calibrator’s bandpass is assumed to be flat. With current bandwidths of the order of 128 MHz, this assumption holds. However, with the advent of broadband systems, such as the EVLA\textsuperscript{3} (Momjian et al., 2009) and CABB\textsuperscript{4} on the ATCA, this will not be valid, (Sault and Wieringa, 1994; Golap et al., 2005).

GPCAL is also applied to the secondary calibrator to determine the complex gains and leakages. Although the Stokes parameters are not usually known for the phase calibrator it is still possible to determine all the parameters if there is good parallactic angle coverage so that the rotation technique described in Section 2.2 can be used. From this technique we can infer the $\zeta^{+-}$ and $\epsilon^{+-}$ parameters (the linear polarisation leakage terms) and thus determine the linear polarisation of the source. The three “problematic” calibration parameters ($\zeta^{++}, \zeta^{--}$ and $\epsilon^{--}$) are determined in the same manner as for the primary

---

\textsuperscript{3}Extended Very Large Array

\textsuperscript{4}Compact Array Broadband Backend
Figure 2.7  ATCA data calibration flow chart. This set of procedures is optimal only when there is good parallactic angle coverage for the secondary calibrator. This figure is adapted from the MIRIAD user’s manual (Sault et al., 1995).
2.4. Deriving Polarisation Images

This leaves two other terms ($\epsilon^{++}$ and $\zeta^{-+}$) still to be determined. These are solved by assuming that the calibrator has zero circular polarisation, thus fixing $\zeta^{-+}$, and by setting the rms gain amplitude to 1, which then constrains $\epsilon^{++}$. The next calibration step is to use GPBOOT, which takes the flux density scale from the primary calibrator and applies it to the secondary calibrator, by comparing the rms gain amplitudes of the calibration solutions.

Finally, the correct calibration is applied to the target source through the GPCOPY task, which copies over the bandpass, gains and leakages, and GPAVER which averages the antenna gains. Once the best calibration solutions have been applied, the next stage is imaging.

2.4.3 Imaging Algorithms

Astrophysical imaging uses a Fourier transform to map the $(u, v)$-data into a sky distribution. The algorithm uses the relationship between the sky brightness $I$, the primary beam pattern $A$, and the visibility $V$ observed with an interferometer. This relationship is given by (Clark, 1999):

$$A(l, m)I(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(u, v) \exp^{2\pi i(ul+vm)} \, dudv,$$

(2.21)

where $l, m$ are direction cosines measured with respect to the $u$ and $v$ axes. The number of individual $(u, v)$ points depends on the number of baselines and the length of observation. With a large number of different visibilities, the sky brightness can be estimated by approximating the right hand side of the above equation as a series of linear operations. This results in a “dirty image”, which is a discrete approximation of the sky brightness, $I^D$ (Clark, 1999):

$$I^D(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(u, v)V'(u, v) \exp^{2\pi i(ul+vm)} \, dudv,$$

(2.22)

where $S(u, v)$ is the sampling function and $V'$ is the calibrated observed visibility at given locations. The sampling function is also known as the “dirty beam”. There is incomplete knowledge of the Fourier transform of the source’s intensity distribution due to the discrete sampling in the $(u, v)$-plane and this process assumes all unmeasured visibilities are zero. Fig. 2.8 shows a typical “dirty” Stokes $I$ map with a strong point source in the field of view. The sidelobes from the point source dominate the image, and make it impossible to detect fainter structure.

For the ATCA data, the grid-and-FFT process is implemented in MIRIAD through the task INVERT. The dirty image is produced through a grid-and-FFT method, though other methods such as a direct Fourier transform or a median Fourier Transform may also be used. It is possible to split the data according to the channels that were recorded or to make the maps combining the channels into a single image in a process called “multi-frequency synthesis” (mfs). The advantage of mfs is that it improves $(u, v)$ coverage and reduces sidelobes, as well as averaging out artefacts that only appear in one channel. However, this technique is not always possible over extremely large bandwidths because
Chapter 2. A Primer on Polarisation

Figure 2.8 This figure shows an uncleaned Stokes $I$ image with a strong point source. The sidelobes from the point source dominate the rest of the image and prevent the other structure from being detected. Before any meaningful analysis can be attempted, the sidelobe structure needs to be removed by deconvolution.

of flux density variation with frequency. One of the important outputs of INVERT is the theoretical rms noise. The noise calculation is based on the system temperature, system gain, integration time and bandwidth stored in the data.

Once the “dirty image” is produced, the effects of incomplete sampling are removed in a process called deconvolution (“cleaning”).

2.4.4 Deconvolution

Incomplete sampling of the $(u, v)$-plane produces artefacts in the dirty image. The convolution theorem is used to interpolate the unsampled area of the $(u, v)$-plane in order to accurately reproduce the sky brightness distribution. The convolution theorem states that the Fourier transform of the dirty image and dirty beam is equal to the convolution of

\[
\mathcal{F}\{I_{\text{dirty}} \ast B_{\text{dirty}}\} = \mathcal{F}\{I_{\text{dirty}}\} \ast \mathcal{F}\{B_{\text{dirty}}\}.
\]

Here, $I_{\text{dirty}}$ is the dirty image, $B_{\text{dirty}}$ is the dirty beam, and $\mathcal{F}$ denotes the Fourier transform.
the Fourier transform of the true source visibility distribution and the Fourier transform (\(\leftrightarrow\)) of the sampling function (the dirty beam, \(B\)):

\[
I_D(l, m) = I(l, m) \ast B(l, m) = V(u, v) \times S(u, v),
\]

where \(I(l, m)\) is the true brightness distribution, \(V(u, v)\) is the true \((u, v)\)-plane distribution and \(S(u, v)\) is the sampling function. Deconvolution aims to remove the sidelobes of the dirty beam. However, the solution is not unique and the end result can be a set of possible solutions from which the most plausible image is made. There are two principal options for deconvolution in radio astronomy: a technique known as CLEAN, and a maximum entropy routine (MEM). Both options have optimal applications which are described in depth in Taylor et al. (1999). A brief overview in this section concentrates on the effects of deconvolution on polarisation images.

**CLEAN**

Högboom (1974) developed the CLEAN algorithm which represents the 2D image as an array of delta functions. The pixel with the strongest intensity is identified and recorded. At this position, a value equal to the dirty beam multiplied by the peak pixel intensity, modified by a gain factor, is subtracted from the image. This process is repeated for a specified number of iterations or until a selected cut-off is reached. The residual image values are then added to the model image created by convolving the set of delta functions with an idealized (normally Gaussian) beam. When CLEANing a Stokes I image all the delta functions would be positive and a positivity constraint can be included in the algorithm. However, with images of the Stokes Q, U and V parameters both positive and negative components are expected.

Subsequent modifications to this algorithm have made speed gains. The Clark (1980) CLEAN algorithm is FFT-based and works very efficiently on large images because it operates mostly on a sub-region of the beam. Another option is the Cotton-Schwab algorithm (Schwab, 1984) which is similar to the Clark method, but uses a more precise subtraction procedure for the peak intensities. An algorithm that is often used is the SDI CLEAN proposed by Steer et al. (1984). They developed this algorithm to deal with large areas of uniform flux density. In the SDI method, any data sample in the residual image greater than some factor times the maximum residual value is redefined as a CLEAN component. Thus, when the residual image becomes very smooth, no artefacts are introduced which could cause spurious stripes to appear in the final image.

**Maximum Entropy Method (MEM)**

As noted earlier, the deconvolution process does not present a unique solution and there are multiple ways of recreating a cleaned image. CLEAN selects a probable image from a set of potentials, but problems may arise because there is no simple equation describing the CLEAN process and the error analysis is quite complicated. A formal error analysis for a variant of CLEAN has been presented by Schwarz (1978). The Maximum Entropy Method selects a positive image which models the data to within the noise level. The
name “Maximum Entropy” was first believed to be related to physical entropy but now the connection is largely discredited. In imaging terms, maximum entropy is a quantity which, at its maximum, produces a positive image with a compressed range of pixel values. There is no clear consensus on the application of this method, as demonstrated by the following sample of publications: Frieden (1972); Gull and Daniell (1978); Jaynes (1982); Narayan and Nityananda (1984, 1986); Cornwell and Evans (1985); Cornwell et al. (1999). In particular, Cornwell et al. (1999) explore algorithms which can be used to compute the maximum entropy value.

CLEAN is particularly well-suited to image reconstruction in radio astronomy. MEM has much wider usage, though is less often used in radio astronomy, possibly because CLEAN is much faster particularly for small images. MEM leaves coupled pixels together which improves the accuracy. Both methods have their weaknesses, CLEAN does not work well for extended emission, producing spurious “blotches”, while MEM does not properly reconstruct unresolved sources located in extended structure, nor does it return the correct value for total flux measurements. Most Galactic polarisation is rather diffuse and extended, and in general the MEM method is preferred, as is discussed in more detail in Section 2.7.

2.4.5 Restoration

After deconvolution, the complete sky image is produced by adding the components model, convolved with a Gaussian beam, to the residual image. The user can specify the full-width at half maximum of the major and minor axes of the Gaussian beam as well as the position angle, measured east from north. If no values are specified then the program searches for Gaussian fit parameters, otherwise the program performs a fit to the beam data.

2.5 Errors in Polarised Images

From correctly calibrated Stokes images, useful polarisation images can be produced. Customarily, three linear polarisation parameters are constructed: the polarised intensity ($P$) image, the polarisation angle ($\theta$) image and the fractional polarisation ($m_p$) image. These are formed from the Stokes $Q$ and $U$ images in the following manner:

$$P = \sqrt{Q^2 + U^2 - \sigma^2}, \quad (2.24)$$
$$\theta = 1/2 \arctan \left( \frac{U}{Q} \right), \quad (2.25)$$
$$m_p = \frac{\sqrt{Q^2 + U^2 - \sigma^2}}{I}. \quad (2.26)$$

One of the more difficult problems with the polarised intensity images is the issue of bias. The polarised intensity $P$, is intrinsically positive, biasing the image and the
2.5. Errors in Polarised Images

Figure 2.9 This cartoon shows the error that corresponds to a particular polarisation measurement. The polarisation “vector” has a \( Q \) and \( U \) measurement, but the associated error can be any of the \((Q,U)\) values in the shaded region.

uncertainties. Bias arises because the measured polarisation of a source (from the observed Stokes parameters \( Q \) \& \( U \)) also contains system noise components, so that the equation, \( P = \sqrt{Q^2 + U^2} \) will be an overestimate of the true polarised signal. Fig. 2.9 shows an example of the polarised signal in the \((Q,U)\) plane, with the uncertainty associated with any measured Stokes signal. Even if the true Stokes \( Q \) and \( U \) values were 0, the errors associated with the observed signal would create a non-zero measured polarised signal.

2.5.1 Debiasing

As we calculate polarisation from the squares of Stokes \( Q \) and \( U \), the probability distribution for the polarised intensity and angle will never be Gaussian. The result is that there is a higher level of noise associated with a polarised signal. Debiasing looks to remove the excess signal to provide the user with a true determination of the polarised signal. Debiasing is problematic when the polarised signal has a low signal-to-noise (S/N) ratio. Typically the polarized signals are weak, particularly from the diffuse ISM, so a careful approach is needed. None of the polarised quantities are Gaussian and because \( P \) is not a direct product of convolution we cannot smooth the polarized intensity data to obtain higher S/N. A solution, from Vinokur (1965), is to debias the polarised signal directly. He derived the results for signals being transmitted in the telecommunications industry and it was applied by Wardle and Kronberg (1974) for polarisation studies in radio astronomy.

The error distribution for polarised intensity is a Ricean distribution given by:

\[
F(p, p_0) = \frac{p}{\sigma_p^2} \exp\left(-\frac{p^2 + p_0^2}{2\sigma_p^2}\right) \cdot J_0\left(\frac{p p_0}{\sigma_p^2}\right),
\]  

(2.27)

where \( p \) is the measured polarised intensity, \( p_0 \) is the true polarised intensity distribution, \( \sigma_p \) is the error of the measured values and \( J_0 \) is the zeroth order Bessel function. Simmons and Stewart (1985), Killeen et al. (1986) and Leahy and Fernini (1989) examined the best estimators for \( p_0 \) given \( p \), and their results are summarised here.
The common estimator considered by all authors was the maximum likelihood, which is the value of $p_0$ that maximises $F(p, p_0)$ for an observed polarisation, $p$. The other common estimator was the worst case scenario where $p_0 = p$. Simmons and Stewart (1985) used two other estimators which proved not very reliable. Killeen et al. (1986) and Leahy and Fernini (1989) were more successful with the method suggested by Wardle and Kronberg (1974), where the estimator is the value of $p_0$ for which the observed polarisation maximises $F(p, p_0)$:

$$\frac{\partial F(p, p_0)}{\partial p} = 0.$$ (2.28)

However, it is also true that an estimator which gives reliable results is simply the first order correction, namely $p_0 = \sqrt{p^2 - \sigma_P^2}$. Fig. 2.10 shows the results of the mean corrected value of $p$, as a function of S/N, using the following integral:

$$\langle \hat{p} \rangle = \int_0^\infty \hat{p} F(p, p_0) dp.$$ (2.29)

The closer the lines are to zero, the better the debiasing solution. Firstly, consider the case where the measured polarisation is taken as the true value (curve a). This solution is the worst case scenario, with a positive bias and significantly more spurious signal added to the polarised intensity. All the other solutions shown in this figure work equally well when the S/N ratio is $> 5$. However, it is for the lower S/N ratios where the performance of the different estimators is critical. The maximum likelihood estimator, advocated by Killeen et al. (1986) is the best estimator when the S/N is extremely low ($< 1$), but it underestimates the true polarisation for some conditions. The estimator asymptotes to 0 at larger S/N ratios. However, of the three useful estimators, it is the slowest to converge at the higher S/N ratios, so it appears preferable to restrict application for situations where the S/N is extremely low. The other two estimator schemes are more generally applicable. Both Leahy and Fernini (1989) and Simmons and Stewart (1985) agree that the Wardle and Kronberg (1974) approximation is preferred because it remains positive and asymptotes to zero the most rapidly. However, the first order correction expression is almost as good, and below 2σ it is actually superior. The IMPOL routine in MIRIAD corrects for debiasing by requiring an input for $\sigma_{QU}$, where $\sigma_{QU}$ is the error in the Stokes Q and Stokes U images, often assumed to be equal to $\sigma_V$. However, IMPOL only allows for first-order debiasing (c in Fig. 2.10). The MIRIAD routine also produces uncorrected images, which are useful in searches for new weak sources, since debiasing isn’t well defined for S/N $< 2$. By not debiasing, one is able to gather statistics for sources detected above a very low S/N cutoff. Finally, note that there cannot be a single “correct” solution, since the underlying distribution of low level polarization is unknown. The choice will depend on what is done with the debiased data.

2.5.2 Errors in Polarised Intensity

After debiasing the polarised signal, uncertainties still remain in polarised intensity, and in this section they are quantified.
Figure 2.10  This plot, taken from Leahy and Fernini (1989), shows the effect of different de-biasing solutions. The $x$-axis gives the S/N ratio of the data, while the $y$-axis shows the difference between the measured and observed polarized signal. The curve labelled (a) corresponds to the measured polarisation curve. (b) corresponds the Wardle and Kronberg (1974) estimator, (c) corresponds to the first order correction described in the text, while (d) is the maximum-likelihood estimator that was advocated by Killeen et al. (1986). The value of each curve is described in the text.
If we ignore debiasing, then the error in $p$ for each pixel ($m$) in the image at a given frequency ($n$) is given by:

$$\delta P_{mn} = \sqrt{\left(\frac{\partial P_{mn}}{\partial Q_{mn}} \delta Q_{mn}\right)^2 + \left(\frac{\partial P_{mn}}{\partial U_{mn}} \delta U_{mn}\right)^2}, \quad (2.30)$$

and using $P = \sqrt{Q^2 + U^2}$ we obtain:

$$\frac{\partial P_{mn}}{\partial Q_{mn}} = \frac{Q_{mn}}{\sqrt{Q_{mn}^2 + U_{mn}^2}},$$

$$\frac{\partial P_{mn}}{\partial U_{mn}} = \frac{U_{mn}}{\sqrt{Q_{mn}^2 + U_{mn}^2}}. \quad (2.31)$$

Substituting from Eq. 2.31, the error in polarised intensity then becomes:

$$\delta P_{mn} = \sqrt{\left(\frac{Q_{mn}}{\sqrt{Q_{mn}^2 + U_{mn}^2}} \delta Q_{mn}\right)^2 + \left(\frac{U_{mn}}{\sqrt{Q_{mn}^2 + U_{mn}^2}} \delta U_{mn}\right)^2},$$

$$= \sigma_{mn}, \quad (2.32)$$

which can be simplified:

$$\delta P_{mn} = \frac{1}{P_{mn}} \sqrt{Q_{mn}^2 \sigma_{mn}^2 + U_{mn}^2 \sigma_{mn}^2}. \quad (2.33)$$

The above equation shows that the error is dependent on both the pixel position on the map and the frequency. If $Q$ and $U$ for a number of frequencies are averaged before calculating $P$, then the equation becomes:

$$\delta P_m = \sqrt{\sum_n \left(\frac{\partial P_m}{\partial P_{mn}} \delta P_{mn}\right)^2},$$

$$\delta P_m = \frac{1}{\sqrt{n} P_m} \sqrt{Q_m^2 \sigma_m^2 + U_m^2 \sigma_m^2} \quad (2.34)$$

$$= \frac{1}{\sqrt{n} \sigma_m}. \quad (2.35)$$

### 2.5.3 Errors in Polarisation Angle

We know from Eq. 2.25 that the polarisation angle is calculated from $\theta = 1/2 \arctan U/Q$, now we can calculate the error for the polarisation angle.

Let us define:
2.6. Errors in Rotation Measures

\[ \delta \theta_{mn} = \sqrt{\left( \frac{\partial \theta_{mn}}{\partial Q_{mn}} \delta Q_{mn} \right)^2 + \left( \frac{\partial \theta_{mn}}{\partial U_{mn}} \delta U_{mn} \right)^2}, \]  

(2.37)

and in a similar manner to the polarised intensity, we can define:

\[ \frac{\partial \theta_{mn}}{\partial Q_{mn}} = -\frac{1}{2} \frac{U_{mn}}{Q_{mn}^2 + U_{mn}^2}, \]
\[ \frac{\partial \theta_{mn}}{\partial U_{mn}} = \frac{1}{2} \frac{Q_{mn}}{Q_{mn}^2 + U_{mn}^2}, \]  

(2.38)

and the polarisation angle error becomes:

\[ \delta \theta_{mn} = \sqrt{\left( \frac{1}{2} \sqrt{Q_{mn}^2 + U_{mn}^2} \sigma_{Q_{mn}} \right)^2 + \left( \frac{1}{2} \sqrt{Q_{mn}^2 + U_{mn}^2} \sigma_{U_{mn}} \right)^2}, \]  

(2.39)

resulting in:

\[ \delta \theta_{mn} = \frac{1}{2} \frac{1}{P_m^2} \sqrt{U_{mn}^2 \sigma_{Q_m}^2 + Q_{mn}^2 \sigma_{U_m}^2}. \]  

(2.40)

and across the whole frequency band, this becomes:

\[ \delta \theta_{m} = \frac{1}{2} \frac{1}{P_m^2} \sqrt{U_{m}^2 \sigma_{Q_m}^2 + Q_{m}^2 \sigma_{U_m}^2}. \]  

(2.41)

In addition, if the Stokes \( Q \) and \( U \) error information is not available it is still possible to determine the error in polarisation angle by using the fractional polarisation. From Wardle and Kronberg (1974) the error in polarisation angle is:

\[ \Delta \theta = \frac{1}{2} \frac{\Delta m}{m}, \]  

(2.42)

where \( \Delta m \) is the error in fractional polarisation.

A final comment on errors in polarised quantities relates to their non-Gaussian nature. It is often preferable to work with the Stokes parameters as they do have Gaussian statistics and the Stokes \( Q \) and \( U \) images may be more straightforward to interpret. This is the approach used with the studies of low intensity polarisation structures in Chapters 4 & 5.

### 2.6 Errors in Rotation Measures

From the polarisation images, other representations of the data are possible, such as the Rotation Measure (RM) map. As described in Section 1.4, the RM map enables estimation of the line integral of the electron density weighted magnetic field. For robust results we need to consider the various sources of error that can occur in RM maps. The RM
is a linear superposition of all the different contributions along the line-of-sight. Under certain circumstances, when there is only one RM across the beam and there is no mixing of emission and rotation, this can be written as:

\[ \theta = \theta_0 + \sum_i(RM)_i \lambda^2, \quad (2.43) \]

where \( \sum_i(RM)_i \) represents a linear sum of all contributions along the line-of-sight. In most cases this can be broken down into:

\[ \theta = \theta_0 + (RM_{\text{source}} + RM_{\text{IGM}} + RM_{\text{Galaxy}} + RM_{\text{ionosphere}}) \lambda^2, \quad (2.44) \]

where \( RM_{\text{source}} \) is the RM intrinsic to the source, \( RM_{\text{IGM}} \) is the RM contributed by the intergalactic medium, \( RM_{\text{Galaxy}} \) is the contribution to the RM from the Galaxy, and \( RM_{\text{ionosphere}} \) is the contribution from the ionosphere. In astronomy, most interest comes from studies of the source, the IGM (Chapter 6), or the Galaxy. The contribution of the electrons in the ionosphere to the resultant RM (Burkard, 1961) varies with the location of the antenna on the globe, as well as time of day and season. Fig. 2.11 shows a map of the distribution of the free electron column of the Earth’s ionosphere at a particular time and date (January 2009, 2100 GMT). The ionospheric effects seen at Narrabri for cm wavelengths is less than 5 rad m^{-2}, (Johnston-Hollitt, private communication), and is usually ignored. The Solar Cycle also affects the amount of rotation produced by the ionosphere. As well as the multiple components along the line-of-sight, there are a number of sources of uncertainty in the RM calculation. Some of these errors are noise, while others occur because of the nature of the polarised radiation.

### 2.6.1 Errors Due to Noise

We have discussed the uncertainties in polarised intensity and polarisation angle extensively in Section 2.5. These errors come from the noise in measurement and any introduced calibration errors in Stokes \( Q \) and \( U \). The signal that is measured can be linearly mapped to Stokes \( Q \) and \( U \) values, which have Gaussian noise. However, the relationship with \( P \) and \( \theta \) is not linear and the errors have a Ricean distribution. This non-linearity is carried through to the RM values. In the complex plane, the measured vector \( \mathbf{P}_{\text{measured}} \) is the sum of the true polarisation vector \( \mathbf{P} \) and an error term \( \varepsilon \):

\[ \mathbf{P}_{\text{measured}} = \mathbf{P} + \varepsilon. \quad (2.45) \]

This equation implies that if the error signal is a significant fraction of the true polarised signal, then the measured polarisation angle will not be meaningful. Hence, if we create individual channel maps as a method of producing a RM map, and the associated error in \( \theta \) is high, then the resulting RM values will also not be meaningful. In RM synthesis (see Chapter 6), the above discussion is not relevant, but as RMs in this thesis are calculated by plotting \( \theta \) against \( \lambda^2 \), it is relevant to include this discussion.
Figure 2.11 The plot shows the electron column of the Earth's ionosphere produced from the data of over 100 GPS receivers positioned all over the globe (shown by the black dots on the map). These receivers provide a snap-shot of the electron distribution at a particular time. The distribution changes quite rapidly over the course of the day dependent on the position of the Sun. One total electron content unit is defined as $1 \times 10^{16}$ electrons/m$^2$. This figure is from http://iono.jpl.nasa.gov
2.6.2 $n\pi$ Ambiguities

One of the deficiencies of plotting $\theta$ as a function of $\lambda^2$ to find the RM is the $n\pi$ ambiguity. As with noise error, this problem is negated by using RM synthesis, but unwrapping was an important consideration for the work on RMs discussed in Chapter 6 and Appendix B and thus will be discussed here. When plotting the polarisation against $\lambda^2$, there is a degeneracy in the number of rotations of the electric vector before it is detected by the antenna. For example, a signal with a polarisation angle of $\theta$, is identical to one with $\theta + n\pi$. This degeneracy can dramatically change the value of the RM and must be considered when doing a least-squares fit to $\theta$ versus $\lambda^2$ plots. In most cases the lowest acceptable value of RM is usually declared the most likely one. It is not possible to fit RMs if there are only two data points as the ambiguity cannot be resolved, and it is recommended to use many data points to counter multiple $\pi$ wrapping. In this thesis, none of the RMs calculated by the author used less than 5 independent data points. Figure 2.12 illustrates the $n\pi$ ambiguity. The measured RM according to the raw least-squares fit is $-26.3 \text{ rad m}^{-2}$. When wrapping is included, the RM fit is excellent, with a value that is much higher at $-137.7 \text{ rad m}^{-2}$.

The most accurate method for estimating the RM is a combination of polarisation angle measurements finely spaced in frequency with a broad frequency coverage. The closely spaced measurements avoid the $n\pi$ ambiguity. For example, if the polarisation angle measurements are separated by 8 MHz, then the RM needed to cause an $n\pi$ ambiguity is around $5000 \text{ rad m}^{-2}$, at 1336 MHz, which is an extremely high value, rarely observed. The second requirement is a number of widely spaced frequency points to enable a robust estimate of the RM. Figure 2.13 shows an example of a source studied in Chapter 6. Some
2.7 Cleaning the Images

In contrast to the strategies used for total intensity deconvolution, a different approach is required when cleaning Stokes \(Q\) and \(U\) images. There are a number of different cleaning options to reduce noise and sidelobes in the image. This section describes some of the issues relating to the correct deconvolution of the Stokes images in \textit{MIRIAD}. Sault et al. (1999) compared the different deconvolution tasks in \textit{MIRIAD}: CLEAN, MOSMEM and PMOSMEM. Both MOSMEM and PMOSMEM are maximum entropy algorithms designed to work on mosaicked images and while the total intensity image must be positive-valued, the Stokes \(Q\) and \(U\) images can have both positive and negative values. MOSMEM deconvolves each Stokes image separately while PMOSMEM deconvolves all images at the same time, which is an advantage when using a non-linear algorithm. A disadvantage with PMOSMEM is that errors from one image can be propagated through the other Stokes images. For example, systematic errors in the total intensity image propagate through
Figure 2.14 The difference in calculated RM between finely and coarsely sampled frequencies for a given region of the ISM. The plot in (a) shows the RM calculated by a large number of closely-spaced frequencies. As the RM is very high the angle changes rapidly between different frequencies. However, in plot (b) the results of four adjacent frequencies have been combined into one measurement and we get a significant difference in the calculated RM. The RM calculated in plot (b) does not give a true representation of the polarisation parameters.

The Stokes $Q$ and $U$ images. Sault et al. (1999) found that for large scale structures, the CLEAN deconvolution both underestimated the large scale structure and produced a number of artefacts in the deconvolution process. MOSMEM and PMOSMEM work on polarised data much more effectively. Sault et al. (1999) found that the best MEM deconvolution algorithm to choose was very much dependent on the type of data being reduced.

In MIRIAD the default number of iterations to deconvolve an image with CLEAN is 250, 30 for MOSMEM and PMOSMEM, although there have been no comparative studies undertaken on the appropriate number of iterations for diffuse polarised emission. It is particularly important to choose correctly for the diffuse polarised emission from the Galaxy, as too many iterations adds noise to the final image. The top panel of Fig 2.15 shows the residual images or models when the default iterations have been completed in MOSMEM (a) and PMOSMEM (b). There is significant structure seen in both residual images, indicating that the image has been overcleaned and emission in Stokes $Q$ and $U$ has been inadvertently removed. A good residual map should exhibit noise only. This is also the case when the CLEAN deconvolution (Fig. 2.15(c)) has been used with 2000 iterations. As diffuse polarised emission, in general, tends to have a lower S/N ratio than other types of radio synthesis images, far fewer iterations are needed for all deconvolution algorithms. The second panel in each set shows when far fewer iterations have been used (10 for MOSMEM (a) and PMOSMEM (b), and 250 for CLEAN (c). In these examples, the effect is less, although it is apparent that structure is still seen in the residual images and the validity of the final image is of concern. By contrast, in the final
2.7. Cleaning the Images

Figure 2.15 This figure examines the effect of the number of iterations on the output of a Stokes Q image. The top three rows of images are the residual images produced by the various deconvolution processes. In (a) MOSMEM is used with three different iterations. In (b) PMOSMEM and in (c) CLEAN is implemented. For each method we show the deconvolved image (bottom row of the figure) using the optimal number of deconvolution iterations.
residual image of each set (third panel from top) only a very small number of iterations has been used in the deconvolution process (2 for MOSMEM (a) and PMOSMEM (b), 100 for CLEAN (c)). For all three algorithms, there is little effect in the final image. However, as maximum entropy tends to work better for extended emission (Sault, private communication), maximum entropy methods are favoured by the author to deconvolve Stokes Q and U maps. By using either MOSMEM or PMOSMEM with two iterations, the data were cleaned and the distortions from the dirty beam removed. Of course, this number of iterations is applicable to ATCA observations of the diffuse ISM structure and deconvolving using \textit{MIRIAD}, other observations with different telescopes and data reduction software packages will be different again. Perhaps the best solution for dirty data is to ensure \textit{full uv} coverage of the area of interest so as limit the number of iterations needed to clean the images in the first place!

A final comment on the difference between PMOSMEM and MOSMEM. PMOSMEM performs a simultaneous deconvolution of all of the Stokes parameters. It effectively searches the Stokes $Q$, $U$ and $V$ images for emission, but only when there is also sufficient Stokes $I$ signal. This can be problematic for interferometric measurements of diffuse emission when there can be polarised flux on small spatial scales, but negligible total intensity. For example, a diffuse region with little structure in total intensity, could contain differently oriented strong magnetic fields producing complex polarised emission.

### 2.8 Next Steps in Interpreting Polarisation Images

Once the images are correctly calibrated and cleaned, the interpretation of polarised structures can begin. Unresolved polarised sources require a different strategy to extended polarised structures.

#### 2.8.1 Polarisation of Unresolved Sources

The \textit{MIRIAD} task, IMPOL does not correctly calculate the polarised flux of unresolved sources at higher frequencies if there is phase decorrelation in the troposphere. This also affects the total intensities in the same way. The reduction in the calculated flux from phase decorrelation can be determined by using the triple product of amplitude\(^5\), which returns accurate fluxes. A comparison can be made:

\begin{equation}
    d = \frac{I_{\text{obs}}}{I_{\text{triple}}},
\end{equation}

where $d$ is the percentage of flux lost due to decorrelation, $I_{\text{obs}}$ is the observed flux and $I_{\text{triple}}$ is the flux calculated by the triple product.

This process is straightforward for all the Stokes parameters, even after debiasing. However, there is still residual flux in the polarised triple product. This additional flux is especially noticeable for weak polarised signals. It is possible to remove this artefact by

\(^5\)Triple product can be expressed as $I_{\text{triple}} = \sqrt[3]{A_1 A_2 A_3 A_4 A_5 A_6}$. 

utilising the fractional polarisation information, which is not affected by phase decorrelation. The true polarised flux is given by:

\[ P_{true} = m \star I_{\text{triple}}. \]  \hspace{1cm} (2.47)

In summary, to obtain accurate polarised fluxes of unresolved sources, use the triple product to calculate the total intensity flux and then combine with the fractional polarisation to obtain the correct polarised flux.

### 2.8.2 Polarisation of Extended Sources

Interpreting the polarisation of extended sources is more complicated than for small diameter sources. It is easy to confuse real structure and artefacts in the Stokes $Q$ or $U$ images. With extended emission the synthesised beam shape is not well defined for the polarised intensity image, formed from the sum of the squares of Stokes $Q$ and $U$, and this introduces artefacts. When unsure whether a structure is an artefact or not, it sometimes helps to examine the same structure at a number of different, closely-spaced frequencies and examine its behaviour. There are depolarisation effects which also can appear in images of diffuse polarised structures. We discuss the interpretation of extended structure in the following chapters.

### 2.9 Summary

In this chapter we have introduced the fundamentals of the polarimetric interferometer, by examining the Stokes parameters, the Poincaré Sphere, and the principles for correctly calibrating a polarimetric interferometer. The work of Komesaroff, Kesteven and Sault in finding the gains and leakages for both an ideal and real world array is summarised. The ATCA is described and the process for reducing data from the raw visibilities into a correctly calibrated image is outlined. A discussion of errors in both polarised images and RM calculations was presented. The final part of this chapter described the steps in analysing and interpreting images of polarised emission.