Measuring the Masses of Galaxies
Edward Taylor <ent@physics.usyd.edu.au>
The University of Sydney/The University of Melbourne

The first and most important thing is to get your head around the mass function, which is the answer to the question: ‘how many galaxies are there that have a mass of \( M \)?

The (stellar) mass function is something you can measure; fine. But why should you care? Well, to the extent that we know anything about dark matter and cosmological structure formation, we know what to expect for the mass function of dark matter halos. And we also know that these dark matter halos are the cradles for galaxy formation.

But as you can see below, the (stellar) mass function for galaxies has a very different shape to what we expect for the cosmological halo mass function. The shape of the galaxy mass function is usually defined in terms of the Schechter function (more on the next page), which behaves like a power-law at low masses, but with an exponential cutoff at higher masses. The halo mass function, on the other hand, is pretty much ‘scale free’—it is pretty much just a simple power law.

In a sense, the defining problem of galaxy formation and evolution is to explain the difference between these two things, by explaining the physical processes that drive, regulate, disrupt, and prevent star formation and stellar mass assembly (remember that galaxies merge!) within the galaxies that sit in centers of large dark matter halos.

In the simplest possible terms, galaxies grow through a combination of continuous and/or stochastic bursts of star formation, as well as through episodic mergers. Remember that more massive stars (stars, not galaxies) burn brighter, bluer, and faster than less massive stars. That means that if you form a whole bunch of stars at once, for the first little while (say, 100 Myr to 1 Gyr), the light from the short-lived massive stars swamps the light from the less massive stars, and that collection of stars will appear as bright and as blue as the few most massive stars there are. As time goes on, and the most massive stars fade and die, the less massive, longer lived population is revealed, and your galaxy will get fainter and redder. Then, if there is another burst of star formation, the galaxy will get bright and blue again, at least for a little while.

So, in the course of a galaxy's lifetime, its luminosity will wax and wane along with its star formation rate, as will its apparent color (as will its size, and just about everything else). In contrast, the mass of a galaxy can (pretty much) only grow. By the same token, the number of galaxies with a mass of, say, $M - \text{i.e., the mass function—can only grow with time, where as the number of galaxies that are brighter than some value } L - \text{i.e., the luminosity function—can either increase (as more stars and/or galaxies are formed) or decrease (as stars die, and galaxies fade). Mass thus provides a good, practical basis for evolutionary studies.}

But there is another reason why mass estimation techniques play such a central role in studies of galaxy formation and evolution. Even though the basic, global observables for individual galaxies—e.g, luminosity, size, star formation rate, metallicity, gas content, etc.—vary by orders of magnitudes, virtually all of these things are tightly correlated with mass. In this sense, mass appears to be a fundamental physical parameter in determining—or at least describing—a galaxy's current state of evolution. Presumably, key information about the process of galaxy formation is encoded form of (and scatter around) these relations.
There are four basic avenues (that I know of) to estimate the masses of galaxies, and these will be the subject of my lecture.

1. **Masses from luminosity** — If you understand (i.e. if you can model) the physical mechanism whereby certain photons are emitted, and you understand (ditto) the radiative transfer of these photons, then you can estimate amount of material needed to produce the observed number of photons.

   The example of this kind of mass estimation that is closest to my heart is estimating stellar masses through stellar population synthesis modeling. This does exactly what it says: you synthesise a population of stars in a way to match the observed spectrum or spectral energy distribution of a given galaxy, in order to estimate the total mass of all the stars in that galaxy. I’ve put just about everything I know about stellar mass estimation in a paper I wrote last year (Taylor E N et al., 2011, *MNRAS* 418, 1587), but I’ve also put together a kind of crib sheet on the next page. The webpages at [http://sedfitting.org](http://sedfitting.org) also provide an excellent resource, including a comprehensive review by C Jakob Walcher.

   This kind of argument is not restricted to stellar masses, though. For example, you can use the flux of the 21 cm line to estimate the amount of cool \((T \sim 10^4 \, \text{K})\) neutral atomic hydrogen gas, or line emission from H\(_2\) or even CO molecular lines to estimate the amount of cold \((T \sim 30-300 \, \text{K})\) molecular gas. At the other extreme, you can use X-ray emission to gauge the amount of hot \((T \sim 10^{7-8} \, \text{K})\) ionised gas in and around the center of a galaxy cluster. Standard techniques for estimating, eg, star formation rates or black hole accretion rates are also based (slightly more complicated versions of) the same kind of argument.

2. **Masses from dynamics** — If you know (or can assume) that a galaxy is in a state of quasi-static equilibrium, then you can use the observed distribution of velocities of some population of test particles to estimate the depth of the gravitational potential well, which is like—but not exactly the same thing as—a mass.

   The crux of this class of argument is that the test particles that you are looking are following closed orbits—or, said another way, that the material is in *virial equilibrium*. This is often short-handed even further to just ‘is virialised’. (If you’re interested in these things, it really is worth spending an afternoon thinking about the virial theorem and its derivation, which you can find on wikipedia.) The bottom line being that by making this assumption, you can relate the kinetic energy, \(K\), to the gravitational potential, \(U\), as \(U = -2K\).

3. **Masses from gravitational lensing** — If you know (even if only in statistical terms) what the shapes of distant sources should look like, and you can measure (again, in statistical terms) the apparent shapes of sources behind some massive structure, then you can attribute the difference between the observed and expected shapes to distortion from gravitational lensing; you can then use this information to build a map of the (2D) projected gravitational potential around that object. I’m going to talk primarily about ‘weak’ lensing, in which you look for correlations in the orientations of many (physically isolated) background galaxies.

   The thing that’s important to realise is that while existing weak lensing techniques are effectively limited to direct studies of cluster-scale masses, they can probe individual galaxies statistically, through ‘stacking’; i.e summing many low-significant detections together.

4. **Masses from clustering** — If you understand the process of cosmological structure formation (on the scales of dark matter halos and larger) to the extent that you can predict the spatial distribution of dark matter halos, and you can measure the spatial distribution of some population, then you can make a statistical argument about the masses of the dark matter halos that that population resides in. This kind of argument is the most abstruse, and the one that I have the least expertise in. But, as just two examples, have a look at Brown et al. (2008, *ApJ* 682, 937) or Wake et al. (2012, *ApJ* 728:46).
Stellar Population Synthesis (SPS): If you know how stars form (specifically, the IMF) and you have models for how they evolve, then 1.-4. provide a recipe for producing a wholly general stellar population. The idea (more below) is then to compare these kinds of models to what you observe for real galaxies, so that you can constrain what kinds of stellar populations do (and, just as importantly, do not) provide good descriptions of your data. In this way, you can say something about the likely characteristics of your observed galaxy’s underlying stellar population.

\[ f_{\text{model}}(\lambda; \text{model}) \, d\lambda = 10^{-0.4 A_V} \frac{t}{t_{\text{form}}} \int_{t_{\text{form}}}^{t} \int_{M_{\text{cut-off}}}^{M_{\text{BD}}} \int_{ZAMS}^{M_*} f(\lambda, M_{\text{ZAMS}}, t', Z) \, d\lambda \]

Building a Stellar Population Library: The key to making SPS fitting useful is constructing parametric families of CSP models. From what you’ve seen, the typical assumptions lead to four parameters: metallicity, Z, dust content, A_V, and the parameter \( t’ \) to describe the shape of \( \psi(t) \). (More on the normalisation of the SFH in a minute.) You could introduce more parameters to accommodate more complex galaxy formation scenarios, but the point is this: by defining the parameter space in which you will place your galaxies, these choices also set the terms by which you will describe your data.

\[ F_X(z; \text{model}) = \int \frac{\lambda X(\lambda)}{D_L^2(z)} \left( \frac{\lambda}{1+z} \right) \, d\lambda \]

From Spectra to SEDs: You know the spectrum for a given model, but most of the time you don’t actually work with a full galaxy spectrum; instead you observe the apparent flux in some broad-band filter set (say, ugriz or BVRI). Let’s consider some the apparent X-band flux for a model placed at redshift z.

4. Dust Extinction/Attenuation: Parameterised by a shape, \( \psi(\lambda, Z) \), and a normalisation factor, \( A_V \). You have to assume a dust attenuation law, \( A(\lambda) \), but note that different ‘laws’ intended to describe different situations. You must specify (or fit for) the normalisation, \( A_V \), which is the total attenuation, in magnitudes, for the V-band (\( \lambda \approx 6000 \) A). Note that the dust is thus treated as a single screen lying between you and the galaxy’s stars.

3. Star Formation History (SFH): From 1. and 2., you know how to describe any population with a uniform age and metallicity; you can construct more complicated stellar populations by combining many SSPs of different ages and metallicities, assuming they are all described by the same IMF. This is commonly referred to as a composite stellar population (CSP). The most common assumption is of an exponentially declining SFH, parameterised by an e-folding time, \( \tau \), and a start or formation time, \( t_{\text{form}} \). It is also most common to assume a single metallicity.

Connecting Models to the Data: Once you know enough to generate ‘mock’ data, then you can ask (in statistical terms) how well your data can only be fit by models with an age of 6-10 Gyr. If your data only fit models with a mass of 6-10 Gyr, then this is your ‘measurement’ of the age of that galaxy. The schematic example below shows how different SSPs lead to different spectral shapes. Each of these models has a different mass-to-light ratio, \( M^* / L \). One you know the right model(s), the you know the right (range of) \( M^* / L \). Then, since \( L \) is something you measure, you have \( M^* \).

Or there’s the easy way: There is a surprisingly tight and linear correlation between rest-frame optical \( (g-i) \) colour, and the value of \( M^* / L \). Part of the reason for this is that variations in metallicity, age, SFH, and dust all shift galaxies away, from the relation. This empirical argument is really just a cheap hack to avoid doing detailed modelling. But, that’s kind of the point: this relation provides a simple and transparent way of deriving meaningful stellar masses based on minimal data.

1. Stellar Evolution Models: This is the basic input for a stellar population synthesis calculations. These models describe the spectra of individual stars as a function of their initial (ie. ZAMS) mass, their age (ie. time since ZAMS), and their initial metallicity. There are a number of models on the market; the biggest uncertainties are in the very complicated situations where evolution is rapid and non-linear (esp. mass-loss along the giant branch).